THE TONAL STRUCTURE OF TAMBURA

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An Exploratory Study of the Tonal Spectra of the Drone
and the related psychoacoustic phenomena

By

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with a Foreword
by Prof. C. R. Sankaran

Dedicated in Gratitude to

Prof. C. R. Sankaran

(Thesis for which the Sangit Acharya Degree of the Gandharva Maha Vidyalaya Mandal was awarded in 1955)

THE TONAL STRUCTURE OF TAMBURA

*FOREWORD

By **C. R. Sankaran**

It is with deep emotions I write these lines. The first meeting I had with the author of this work nearly a decade ago is still fresh in my mind. Ours has been a memorable and the most happy association and collaboration since that period, in a work so near to our heart.

Dr. CHAITANYA DEVA has so generously referred to my accepting his personality **in toto**. But I must here improve the statement by suggesting the modification that this has been mutual with us. I, therefore, consider it a rare privilege to introduce this (in every sense of the term) pioneering work particularly to the lovers of Art-Symbolism in addition to the serious students of psycho-acoustics of Music and Speech, to whom the work is primarily and ostensibly intended. With regard to my apparent strictness in methodology to which a generous reference has been made by the author of this fine work in his preface, I make bold to declare here my cherished conviction that rigour is of the essence but not an end; "like cleanliness, it is a means to an end, the end being health; and when the technicalities of rigour have bec[...] known, their obtrusive objectionabl[...] well be omitted if the spirit of [...] preserved". (Cf. E. A. MILNE, **Cosmology And The Christian Idea** Oxford, At the Clarendon Press 19[...] I would say then with the poet – **naiva sastram naca kramah**.

In the last chapter of the present work, there is an exposition of the theory of the alpha-phoneme at a refreshingly aesthetic level. This is beautiful in itself— "our inmost in the inmost way" expressed. [1] I consider this assessment as not only charming, but I go still further now and assert that this assessment represents truly the nearest to my own theoretical conception.

The term 'non-temporal wedge', as the author of this work has happily chosen to describe it, best fits the psychological experience, known to the Indiam mind through the familiar term **rasānubhava**. [2] The sense of the deeper links between various levels of sensory experiences, of the complexity of the entire

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* This foreword in a slightly modified form has already been published under the title, "Recent trends in the researches on the psychophysics of Indian Music", **BDCRI** 19. (1958) pp. 125-33.

1) Dilip Kumar ROY, **Sri Aurobindo Came to Me** (Sri Aurobindo Asram, Pondicherry, 1952) K. D. SETHNA's Introduction. iv.

2) Perhaps the term **rasānubhūti** might be preferred here, bearing a maturer emotive import as Dr. Sudhir DAS GUPTA suggests.

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structure, and, above all, of interdependent inter-relations, not merely among ideas, the products of thought, but in the very process of **knowing** should necessarily make a thoughtful observer who disregards the time-honoured and convention-respected 'psychological security', a questioner (more often an inconvenient one) – for precisely the very reasons which imply a dangerous living – than a systematizer, provocative rather than dogmatic. I prefer, therefore, a not-rigidly-compartmentalized way of asking questions, which are indeed more important than the answers. So, I am happy all the more to find as the guiding motive of the present author's work, this very spirit of thinking and living which distinctly imply that **one should value crystallized systems less and an ever challenging enquiry more.** For, it is clear that the area of the unprovable is encroaching on the proved.

We are, however, aware (sometimes most painfully too!) that still many dislike the tentative and the exploratory. They want systems and answers (readily cooked!) if only to refute them. But our choice is deliberate to seek an audience among those who no longer expect to find the 'whole truth' (if that can be done at all!) arranged neatly and labelled as specifics for all the challenging problems. We exhort such an audience to be ready as never before to value those reachings out of the mind for what lies on the peripheries of knowledge and experience. Our conviction is strong and deep of the **transcendency** of 'something' to the merely 'pragmatic, conventional and empirical, and of the inexpressible comfort (clearly distinguished from the empirical one which goes by this very name in our miscalled normal and the surface-state of a fancied 'psychological security'!) [2a] and in-

(*Foot Note No, 2 Continued from page ii*)

In essence, it is what has been so tellingly and happily designated by ABHINAVAGUPTA as the **vi'srānti dhāman**– the absolute of our experience here (of. also **infra** footnotes 5 and 8)— [cf. V. A. RAMASWAMY SASTRI, Studies in Dhvanyāloka, **BDCRI** 17. 223–30, 1955; cf. too in this connection, **The Dhvanyāloka of Ānandavardhānācarya** with the Commentary (called **Locana**) of ABHINAVAGUPTACĀRYA, 28, Kāvyamālā 25 Bombay, 1911 and **Dhvanyāloka** by **Ānandavardhana and Locana** by **Abhinavagupta,** Ed. S. KUPPUSWAMI SASTRY, etc., 259, Madras, 1944]– the last effective term, as it were, of an elegant equation, stressing and simultaneously pointing towards the culminating 'one-pointed' experience of **timeless** 'awareness'.

2 a) Obviously the **static** concept of security which "may be pictured by thinking of the oyster inside its shell, the, frightened person behind his neurotic defenses, or prewar France behind Maginot Line" is referred to here. cf. S. I. HAYAKAWA, The Fully Functioning Personality, **ETC.** XIII. No. 3 Spring 1956. p. 169, while A. S. EDDINGTON speaks only of a healthy "dynamic, inner security" at the conclusion of his Swarthmore Lecture 1929. (**Science and the Unseen World**, London, George Allen and Unwin, June 1930, p. 56) when he says "Religion for the conscientious seeker is not all a matter of doubt and self-questionings. **There is a kind of sureness which is very different from cocksureness**". (Emphasis mine).

The "dynamic, inner security" may be compared, as Dr. P. C. GANESHSUNDA-

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ward strength, as we serenly contemplate in calm quietude truth realised into Being by the human will.

Such a concern with the 'ultimates' for other aims is perhaps quite unnecessary and may very well prove to be excessive too, although we readily agree (as it is inevitable, in obedience to the logic from **within**, which is based upon facts of an order having their abode in the constitution of the Mind) that even the most exalted mystic some time or other has to touch the physical. [3]

But only our emphasis has ever been in seeking and seeing things 'through symbolic eyes enlarged by cosmic wandering'. [3a] Such an experience is liberating, being the "sower

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RAM rightly observes, to the "dynamic stability" of a rotating top as against the collapse of the stability of the top in the absence of the dynamic state of rotation. (cf. Also for a discussion on the stability of rotating tops and related problems, P. POLUBARINOVA-KOCHINA, **Sophia Vasilyebha Kovalevskaya – Her Life and Work,** Foreign Languages Publishing House, Moscow, 1957, p. 37 ff and 57 ff.).

The "dynamic, inner security" is a 'pure **intellectual conviction**' raised to the status of an **emotion** Cf. "Probably when the subject is thoroughly examined, **conviction** will prove to be one of the intensest of human emotions, and one most closely connected with the bodily state". **Alethetropic Logic**, A Posthumous Work by Sir Almroth WRIGHT, presented by Giles J. ROMANES, London, William Heinemann Ltd., 1953. Chapter. VIII. On further Points Relating to Believing. p. 44. note. 1. Cf. Also Walter BAGEHOT, On the Emotion of Conviction. **ibid**. Appendix 16. pp. 331–334.

3) Here it is clear that distinctly I mean **not** the so-called **accomodation** of 'idealism' to 'realism'. S. Kuppuswami SASTRI, **Compromises in the History of Advaitic Thought**. 10 (Kuppuswami Sastri Research Institute., Madras, 1946).

3a) Cf. "For most of us most of the time, the world of everyday experience seems rather dim and drab. But for a few people often, and for a fair number occasionally, some of the brightness of visionary experience spills over, as it were, into common seeing, and the everyday universe is transfigured. Though still recognizable itself the Old World takes on the Quality of the mind's antipodes". (Aldous HUXLEY, **Heaven and Hell**, Chatto and Windus, London, 1956, p. 17) Cf. also, "Augustine said that God created the universe new everyday : and to the living, emotional soul this is true. Every dawn dawns upon an entirely new universe, every Easter lights up an entirely new glory of a new world opening in utterly new flower. And the soul of man and the soul of woman is new in the same way, with the infinite delight of life and the ever-newness of life. So a man and a woman are new to one another throughout a life-time, in the rhythm of marriage that matches the rhythm of the year". **Sex, Literature and Censorship Essays** by D. H. LAWRENCE, Edited by Harry T. MOORE. William Heinemann, 1955. p. 250)

of all true art and science" (EINSTEIN), which is most certainly present in persons of maximum creativity and "psychological health".[4] For, 'it is in no way incompatible with science, and it does not take the place of science. Nor can science take its place or negate it. Semantically it is clean. Humanly it is finely and finally satisfying'.[5]

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4) 'Self Actualizing People', **Personality**: Symposium No. 1, 1950 as quoted by Charles MORRIS in, Comments on Mysticism and Its Language (Ed. S, I. HYAKAWA, **Language, Meaning and Maturity** 301, New York, 1954).

5) Charles MORRIS, **ibid.** 302

It must be remembered here that the quest after the 'ever-receding beyond' we give expression to throughout our exposition of the alpha-phoneme theory reftects a deep experience, echoing distantly, as it were, the beautiful passages in the **Brhadāranyaka Upanisad,** III. 6 : 1 and III. 8: 3-8 **Brhadāranyaka Upanisad,** Ramakrishna Math, Madras, 1951, pp. 222 and pp. 244-247).

The pursuit after the 'ever-receding beyond' in the manner indicated here has been already implied by the statement "Time as **Eternal-Now** transcends all empirical determinations". (C. R. SANKARAN, A Philosophical Analysis of the Alpha-Phoneme Theory in Relation to the Problem of Speech-Structure, **BDCRI**, vol. 14, 1952, p. 97).

Also, the different levels in the 'ever-receding beyond' are parallel to different levels in our geometrical thinking as well as to the eight levels of **sphota** of BHARTRHARI [(cf. C. R. SANKARAN, **ibid.**, Addendum II, p. 100). cf. also C. W. LEADBEATER, **The Masters and the Path,** Theosophical Publishing House, Adyar, Madras, India, 1927; 2nd ed., p. 339].

The 'end-point' of our ever widening search after the 'ever-receding beyond' is, as it were, a purely **advaitic** experience of the **Timeless-Time,** which first destroys the 'befores and afters' (and their identities as well!) of all levels of consciousness, and finally vanishes itself, having now fulfilled its function as an indicator (**upalaksana**), submerging, so to say, in the 'Ultimate Reality' (cf. The following **Vārtika** for which I am indebted to Sri T. S. SRINIVASA SASTRI of the Sanskrit Dictionary Department of the Deccan College Research Institute) :—

Avidyāstamayo moksah
Sāca bandhah udāhrtah
Nivrttirātma mohasya
Jnātatvenopalaksitāh.

(Cf. **supra** footnote 2 and **infra** footnote 8) Cf. too. Taylor's following significant observation in this connection :- "Somewhere behind all evolutions and supplying all with 'material' and 'driving force' there must be the strictly eternal". TAYLOR, **Evolution in the Light of Modern Knowledge,** as quoted by J. J. Van der **Leeuw, The Conquest of Illusion** London, George Allen and Unwin Ltd., 1951, p. 98).

Obviously, this final dissolution itself is in order; for "knowledge (itself) is a form of ignorance". (Cf. Aldous HUXLEY's Fore-

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I believe therefore, that it is no exaggeration to describe the present work more or less in the very words of Christmas HUMPHREYS [6] originally applied to the much renowned Dr. D. T. SUZUKI who is more than a mere scholar, being one who dwells in the higher stages of consciousness, as yet another ardent attempt made in recent times " to seek for the intellectual symbols wherewith to describe a state of awareness which lies indeed beyond the intellect ".

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word to J. KRISHNAMURTI's **The First and the Last Freedom,** London, Victor Gollanz Ltd., 1954, p. 18 ; cf., too, **Brhadāraṇyaka Upanisad,** loc. cit., III. 5:1, p. 219 **Is'opanisad,** verses 9, 10 and 11, Sri AUROBINDO, **Eight Upanishads,** Sri Aurobindo Ashram, Pondicherry, 1953, pp. 8-11; and C. R. SANKARAN, op. cit., Addendum I, p. 104.

Profoundly, Dr. G. WERNER (Department of Pharmacology, Cornell University Medical College, New York) in a letter to me (dated 23rd July 1956), has observed:– " No meaningful question can be asked without knowing at least one possible answer in advance ". The ' ever-receding beyond ' of ours, with all its implications as stated above, is the " answer in advance ", given by us here. Cf. "....it is a profound truth that when once we can really ask we also know" (J. J. Van der **Leeuw ibid**., p. 91, For, to illustrate this " answer in advance " in modern Physics, is it not true that " at the basis of the Ergodic Theory, there will always occur in a closed system at some time, a decrease of entropy after it has sufficiently increased " [quoted from Dr. G. WERNER's letter referred to above. See also Ernest H. HUTTEN, **The Language of Modern Physics, An Introduction to the Philosophy of Science.** London:– George Allen and Unwin Ltd., New York : The Macmillan Company 1956, p. 226; again on page 149 we read, " Entropy does not specify a unique direction of time; it allows us merely to order events, or states of the system, within a limited time interval. Even the statistical interpretation of thermodynamics cannot escape the idea of reversibility inherent in our basic conceptions ". cf. too p. 152, " once entropy has increased, it must decrease again, in a closed system. The irreversibility, or the increase in randomness, occurs only within a limited, though very long, time interval "], disturbing our usually held conception of " time-arrow "? Here, one may also profitably remind oneself of the " negative energy levels " A. S. EDDINGTON speaks of, in his **Fundamental Theory** (Cambridge at the University Press 1946. IVth chapter, Gravitation and Exclusion Sections 42 pp. 83–85.) But it must be remembered here that a possible answer is not a " definite answer " cf. Max PLANCK, **Scientific Autobiography and other Papers** p. 139. see also fn. 7. infra.

6) Editor's Foreword to Daisetz Teitaro SUZUKI'S, **Essays in Zen Buddhism** (London, Rider and Co., 1950).

Even from the purely empirical standpoint, the best introduction to our way of living and thinking (if I may venture to describe it thus) as deeply reflected in the work we have undertaken, would be, I think, Philip M. MORSE's, Acoustics and Basic Physics, **JASA** 27. 213–6, 1955.

The attempt to seek such an **absolute** is no new one at the limits of Science, which properly belongs to the realm of Scientific Philosophy or Philosophy of Science as it is sometimes called. For, "The Theory of Relativity, too, is based on something absolute, namely, the determination of the matrix of the space-time continuum; and it is an especially stimulating undertaking to discover the absolute which alone makes meaningful something as relative .... Our task is to find .... the absolute, the universally valid, the invariant, that is hidden......The theory of Relativity confers an absolute meaning on a magnitude which in classical theory has only a relative significance: the velocity of light. The velocity of light is to the theory of Relativity as the elementary quantum of action is to the Quantum theory: it is its absolute core. In this connection, it turns out that a general principle of classical theory, the least action principle, is also invariant with respect to the Theory of Relativity; accordingly, the quantum of action retains its significance in the Theory of Relativity as well". 7

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7) Max PLANCK, **Scientific Autobiography and other Papers** 47-8, Trans. from German by Frank GAYNER (London, 1950). For **continum spatii et temporis est absolutum** vide A. EINSTEIN, **The Meaning of Relativity** 54 (Methuen, 1950) [cf also, B. ABRAMENKO, On Dimensionality and Continuity of Physical Space and Time, BJPS, IX, pp. 89- 109, 1958]

In the letter referred to in footnote 5 **(Supra)** Dr. G. WERNER observes– " induction is not a Hypothesis – creating process; it is statistical inference with reference to an Hypothesis already existing. Any Hypothesis, we create......by means of a deduction " (cf. Ernest H. HUTTEN, **op. cit.**, p. 226). It is also significant that EINSTEIN had declared his distrust of statistical interpretation in Quantum mechanics, in unambiguous terms (Max BORN, **Natural Philosophy of Cause and Chance**, Oxford At Clarendon Press. 1949 p. 123). Also Max BORN, EINSTEIN's statistical Theories. **Albert-Einstein Philosopher Scientist**. The Library of living Philosophers, New York, Tudor Publishing Company, 1951, p. 177, where he refers to the best known paper of Albert EINSTEIN, B. PODOLSKY, N. ROSEN: " Can Quantum Mechnical Description of Physical Reality be considered complete? **Physical Review.**, 47, 1938 p. 777. From this point of view cf. Hans REICHENBACH, **op. cit.**, p. 170 cf. too," Her (nature's) fundamental laws do not govern the world as it appears in our mental picture in any very direct way, but instead they control a substratum of which we cannot form a mental picture without introducing irrelevancies. The formulation of these laws requires the use of the mathematics of transformations. The important things in the world appear as the invariants (or more generally the nearly invariants, or quantities with simple transformation properties) of these transformations. The things we are immediately aware of are the relations of these nearly invariants to a certain frame of reference, usually one chosen so as to introduce special simplifying features which are unimportant from the point of view of general theory." (P. A. M. DIRAC, **The Principles of Quantum Mechanics** from the Preface to the First Edition p. vii. Reprinted lithographically in Great Britain At the University Press, Oxford from sheets of the Third Edition. 1956)

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The **alpha-phoneme** theory, addressing itself to the temporal ' between ', perhaps may give satisfaction in this quest for the **absolute**. For, we now conceive of an ' interval ' which is " an ordering element in that it connects series which in themselves are unconnected". 8 The staggering consequence is the unassimilability of **time** to Cantor-Dedekind type of continuity to which we have been all along forcing ourselves to adhere, despite its inadequacy and unsatisfying nature. 8a In the newer freedom we have thus found to be so liberating, we emancipate ourselves from an intellectual apathy and torpor (I mean, not being alert intellectually) to which even some assiduous minds have been bound uptil now. We are now on the threshold of a continuum of a higher (i. e., less restricted) type than that of Dedekind. This Veronesean or Hilbertian continuum is perhaps the most suited key also for unlocking the mysteries of the much vexed **s'ruti** problem. It is not proper that one should enter into this fascinating question in a foreword to a preliminary work in this direction. So only whetting the appetite of the interested reader at this stage (for which

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Here the recognition of ' no one consistent scheme of interpretation of physical phenomena ' (Robert Andrews MILLIKAN, **Evolution in Science and Religion** 27, New Haven, Yale University Press, 1930), is the first step in the direction of a unified field theory, the context of reference to which [in footnote 23, pp. 405-6 of the paper, Time and Speech-Structure, by C. R. SANKARAN and P. C. GANESHSUNDARAM, **BDCRI** 12] is to be specially noted with all its deep significance and implications.

8) C. R. SANKARAN, A Note on the Epistemological Significance of the Alpha-phoneme and the Alpha-phonoid Theories, **BDCRI** 14 No. 2. 120, 1952 (of. **Supra**, footnotes 2 and 5).

As C. D. BROAD in **The Mind and Its Place in Nature**, Preface, viii (London, Kegan Paul, Sixth Impression 1929) pertinently observes, we should not " confuse the Author of Nature with the Editor of **Nature** " and it is not at all proper " to suppose that there can be no productions of the former which would not be accepted for publication by the latter ", conditioned by empiricism. Absolutely there is no reason to believe this empirical dogma, so to say.

8 a) In the sub-atomic world, the conventional temporal and spatial notions are to be got rid of altogether. " In the case of time it seems possible for us to **live** in a Universe where the ordinary temporal relations do not hold ". (Aldous HUXLEY, **Do What You will** Pascal, Chatto and Windus, London, 1949, p. 263. Cf. also C. T. K. CHARI. **Is Time Matrizable ? Procedings of the Indian philosophical Congress** 1951. pp.129-141, especially p. 140, footnote. 40, where a generous reference is made to my theoretical conceptions. cf., too, C. T. K. CHARI, A Note On Multidimensional Time, **The Brit., Journal for the Philosophy of Science,** Vol. 8. 1957. pp. 155-8, and especially p. 158). Hans REICHENBACH in his posthumous work **The Direction of Time** discusses the qualitative, or topological, attributes such as order and direction of time cf. Maria REICHENBACH, Preface, vi, H. REICHENBACH. **The Direction of Time**, University of California Press, Berkeley and Los Angeles 1956). cf. also B. ABRAMENKO, **Op. cit.**, referred to in ft. nt. 7 above.

I crave his indulgence), I leave it at that with an assurance that it is hoped both by us to continue the work in that direction more fully, God willing, later. 8b The path is clear and now it will prove only as yet another leap into a fresh adventure. Until then, waiting is quite unavoidable and, therefore, I enter now, in the meanwhile, into an evaluation of the present work at somewhat technical levels in the rest of this foreword, as **an amateur** [" in the strict and noble sense of the word–that is, a **lover** of the subject " even as WHORF was in linguistics, cf. Garrett HARDIN, Meaninglessness of the word Protoplasam, **ETC.** XIII No. 3 Spring (July) 1956. p. 199] in the field of Psycho–Acoustics.

It is evident from the work that the main aim of the author was in the beginning of the investigation, at any rate, the study of the acoustic–structure of the **tambura.** It is, threfore, gratifying to note that sufficient data for harmonic analysis towards this end, have been satisfactorily gathered and neatly presented in the work.

It is to be appreciated that the author was conscious of the serious limitations his task had imposed upon himself, and therefore, confined only to the twelve ordinates giving six harmonics and at the same time indicating approximate results by calculating the inharmonics by the method of weighted mean, due to HERMANN and SCRIPTURE.

It is gratifying to observe that in the present work the subjective tones have been calculated from the non–linear transformations as given by FLETCHER.

It is highly commendable that in this exploratory and piloting work, the following points have been brought into great relief :–

i) **Tonal memory** and **tonal retro-relation** in melodic music.

ii) The unique experimental work with reference to the string–bridge contact which is so significant from the point of view of the physical cause for the tonal structure of the instrument, being more complicated than Prof. C. V. RAMAN had earlier supposed it to be.

iii) The admirable clarity in regard to the discussion of ' subjective tonal spectrum ' in Chapter IV and the pleasant defence of the use of the term ' subjective tone ' as against the hard–boiled behaviourist's objection to it.

iv) The **approximate estimate of the** subjective–tone spectrum.

v) The distinction between **tone** as a ' psycho–physical reality '– and a **note** as a symbol indicating a musical relation of tones. For a **note** is not an objective reality. It denotes a relation in the ' tonal movement ' of any particular stage only as a mental convenience.

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8 b) Since writing this Dr. B. Chaitanya Deva and I have published a paper on the subject in the **BDCRI**, Vol. 18, pp. 192–204, which is followed by Dr. Deva's, " The Vibrato in Indian Music " in **Acustica** Vol. 9 (1959) pp. 175–80.

vi) The details of the analysis of the **external** (' acoustic ') **stimulus** and the ' neuro-physiological response '— correlates thereof, as beautifully indicated with admirable clarity in Chapter V, leading in their wake to the need for a recognition of the " non-temporal wedge " as it were, in the **fulness** of ' experience ' and as a naturally expected consequence, the most refined aesthetic and philosophical assessment as well of the nature of **time** at various psychological levels.

Above all, it is to be noted that the profound conclusion that " the state of aesthesis (**rasanubhava**) is **timeless** " and that " even in conscious behaviour attention is not a continuous process, " is most arresting. The use of the most happy term ' inter-phenomenon ' in this crowning context, originally due to REICHENBACH, but extended and used in a special sense consistent with our thinking, it is again to be noted, is most appropriate, suggestive and illuminating, as a fitting close to this remarkably pioneering work in this field, showing the underlying spirit of the entire original investigations embodied in this brochure, although the term ' inter-phenomenon ' was originally used by REICHENBACH and his successors to the day in a purely empirical sense (Cf. Ernest H. HUTTEN, **The Language of Modern Physics,** 1956, p. 195); for, to us the **inter-phenomenon** is the " unobservable " contradistinct from the " unobserved " of the empiricists. (Hans REICHENBACH, **Philosophic Foundations of Quantum Mechanics,** University of California Press 1946. pp. 21 and 177; Max BORN, **Natural Philosophy of Cause and Chance,** Oxford, at the Clarendon Press 1949. p. 107).

Only I have a personal preference here for the term ' non-conscious ' to the author's ' collective [un]-conscious ' (a term apparently due to C. G. JUNG, I think), although I am convinced that both the author of the present work and I perhaps mean the same thing (if this can be called a thing at all, in the common parlance, using the ordinary language !).

Be it noted that the author himself here switches off quickly to the term ' non-conscious ' from his original '(collective) [un]-conscious '. I, for one, would not waver between the two. I prefer as I have said already, the ' definite language ' here for once (I trust that this will not be attributed to me as a strange inconsistency and paradox, for this preferencc is only here, probably an apparent exception to the general rule !)- viz., the term ' non-conscious '. Even this term is not free from ambiguity. For, it may also by some stretch be taken to mean " mental processes which are not attended to by consciousness " altogether.

I, therefore, for my part, am inclined to say with Sri KRISHNA PREM [9] that only "we can talk of a sub- or super-conscious mind, meaning by those terms mental processes that are sustained in consciousness below or above the level at which it is normally focussed, processes which are not attended to by nor-

9) **The Yoga of the Bhagavadgita** 197 (London, 1951, 3rd edn.)

mal consciousness, but we can not talk of an unconscious mind, for that would have no meaning". Undoubtedly I know for certain that the author of the present work knows all this. But the difference lies in our temperaments (which have been accepted with mutual reverence, as have been already remarked earlier with sufficient stress, I think) and he sees, doubtless, no harm in adhering to the term 'unconscious'. Only I sense a threatening danger here for the unwary reader of the work who may be caught up in this (what appears to me as an) unfortunate term which may, therefore, prove to be a veritable trap indeed, even when all is said and done, as I have tried to indicate above.

The work is bristling with rich suggestions, some of which I wish merely to indicate in conclusion :—

i) Further controlled experiments in a well-constructed anechoic room are necessary.

ii) The sub-harmonics of the tambura require a detailed investigation. It is a matter of deep personal satisfaction to me that my other associate in research Dr. P. C. GANESHSUNDARAM is now studying the problem, approaching from the point of view of the modulation spectrum, in the Phonetics Laboratory of the Deccan College Research Institute, where originally the author of the present work carried out his investigations embodied here.

iii) There is now a dire necessity of repeating Dr. CHAITANYA DEVA's work on the string-bridge contact of the tambura with a mechanical plucker and bridges of varied slope, with a view to eliminate all personal errors.

iv) Also, further intensive experiments with exploring (probe) tone are a matter of sheer necessity.

v) The preliminary studies of the tambura as represented in this work must be followed by an intensive study of all our Indian musical instruments.

vi) Above all, pilot experiments are indicated now, more than ever before, to determine the connecting links which will bridge the 'conceptual' and the 'symbolic' levels touched by the alpha-phonoid and the alpha-phoneme theories respectively, although the relation and the relata of these two theories, in the very nature of things, can never find an unambiguous translation in the experimental realm – the gulf between the theories, therefore, and the realm of experimental observation being logically ever unbridgeable in this sense.[10]

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10) In this connection KANT'S demonstration of the incorrectness of the procedure 'of clothing "metaphysical intuitions" in the forms of empirical knowledge ....' is to remembered. P. DEUSSEN, **The Philosophy of the Upanishads** ix (Edinborough, 1906)

It is useful to remember also here that space-time of the special theory of relativity must be regarded as something more than a coordination of "perspectives", for propositions concerning its relata and relations do not admit of an unambiguous translation into the spatial temporal framework of experimental physics'. C.T.K. CHARI, An Epistemological Approach to the Special Theory of Relativity, 2. Einstein's Mechanics. **MIND** 47. No. 188, 552.

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I consider it a very happy augury that such a brilliant work on the tambura has been awarded the Sangīta Ācārya (Ph. D. in the theory of music) degree by the Gandharva Maha-vidyalaya at Bombay. I end now by heartily congratualating both the worthy receipient and the discerning donor of this unique and well-deserved distinction. I thank from the very bottom of my heart the author of the work for having requested me to write a foreword to his fine work, the first of its kind. For, I consider that no joy equals the one of introducing an original work such as this to the discrimination of the adventurous readers, endowed with an unabatable intellectual curiosity. Indeed here is offered a rich mine of information, a veritable 'feast of reason and flow of soul'. Adopting here unhesitatingly Madame H. P. BLAVATSKY's telling description of ANNIE BESANT, in short, I call the author of this important work as the " one, the only one " I know of.

Phonetics Laboratory,
Deccan College Research Institute,
Poona - 6.
25th September 1958.

— C R. Sankaran.

(*Continued from page No. xi*)

It may be pointed out here that the Alpha Phonema and the Alpha Phonoid constitute respectively " a transcendental unity of Being Beoming-or rather a Being that is in its essentiality an eternal Becoming, 'a sort of fundamental blissful dualism in unity' (**The Life Divine** I,225), " Paul **Colaco**, S, J., Sri Aurobindo- A Philosopher of Reconciliation. **The Modern Schoolman**, A Quarterly Journal of Philosophy, Vol. 28. November 1950 to May 1951. p. 297. Saint Louis Uuiversity. Cf. also here P. C. GANESHSUNDARAM, The Process Existence concept and the Structure in Speech, **BDCRI**. 18. (1957), pp. 205-214. Cf. too my popular lecture **" Open, oh ' Sesame' or Speech and Speech-Activated Devices,"** to be published. In such a symbolic representation of an ultimate experiencing, the observer is " tightly coupled with the observed phenomenon " in more than an empirical sense of the phrase. Cf. E. Colin CHERRY, Roman Jakobson's Distinctive Features as the normal coordinates of a language, **For Roman Jakobson** 61-62 (Mouton & Co, 1956). Cf. also B. Chaitanya DEVA, Action as a measure of accent, **BDCRI** 18.285 ft. nt. 18 (1957).

Preface

The work reported in this thesis is the prelimary part of the planned studies in speech and music, being carried out in the Phonetics Laboratory, Deccan College Research Institute, Poona, under the guidance of Prof. C. R. Sankaran.

I began the study of the psychology of music about a decade ago. Since such a study had not been pursued in any systematic manner in modern musicology, it was not an easy thing to know of a point of departure. Vague and unco-ordinated ideas which, if properly arranged, could form a very good plan of study, presented themselves to me. It was at such a juncture that I met Prof. C. R. Sankaran who encouraged me and gave me the valuable help of his erudition and strictness of methodology. The result of our discussions – and I must here not fail to acknowledge my debt to him for accepting my personality in all our exchanges- was the papers, "Postulational Methods and Indian Musicology" and "A Note on the Drone in Melodic Music". These two suggested to me that the psychology of the drone would be the best point to begin the study of the psychology of our music.

A fairly detailed discussion of the psychology of the drone in our music was presented in, "Psychology of the Drone in Melodic Music" and the effect of the drone on the development of our musical forms was discussed in "The Emergence of the Drone in Indian Music". The present thesis is the report of the acoustical analysis of the tambura. A comprehensive perusal of all these would reveal how fruitful such a research has been. It has opened up great and enchanting vistas for further research in speech and music. I am, indeed, happy that this humble work of mine has been the beginning of such brilliant researches of Dr. P. C. Ganeshsundaram in speech and music and Sri V. R. Patwardhan in musical instruments.

I must express my profound indebtedness to the following:

The Director and other authorities of the Deccan College Research Institute, Poona for providing me with the facilities and scholarship for this research in the Phonetics Laboratory of the Institute.

The Librarian, Deccan College Research Institute, for all help in the matter of obtaining readily the literature in connection with this work.

To Dr. P. C. Ganeshsundaram, Scientific Assistant, Phonetics Laboratory, Deccan College, Dr. A. D. Taskar, formerly a student in the Laboratory and now Statistician, Nutrition Research Institute, Coonoor, Sri V. R. Patwardhan and Sri Sripad Rao Savle, Laboratory Assistant, for unstinted help in the experimental work, laborious calculations and the preparation of the thesis.

To Sri Vishnubhau Dhalgaonkar (Bapu) Musical Instrument Maker, Poona for showing me the making of the tambura bridge which is a closely guarded trade secret !

To the Principal, N. Wadia College, Poona and Prof. M. G. Mahajan, former Head of the Department of Physics of that College, for giving me facilities for photographing the tambura bridge. I am particularly grateful to Sri C. V. Bapat, Cancer Research Institute, Bombay for the excellent photography of the tambura bridge shown in Fig. XIV.

To Sri H. V. Modak, Dept., of Physics, N. Wadia College, Poona and to Sri Shaikh Zainuddin Ansari, Deccan College for the experimental set up and the photography of the tambura string shown in Fig. XIV a.

I must also acknowledge my indebtedness to Profs. E. B. Newman (Harvard) and E. G. Wever (Princeton) for help and guidance in the study of melodic integration and auditory mechanism.

Above all, I am greatly indebted to Prof. C. R. Sankaran, Reader in Charge of Phonetics Laboratory, Deccan College Research Institute, Poona for constant encouragement and sympathy in all my work.

I wish to record finally my thanks to the President and other authorities of the Gandharva Mahavidyalaya Mandal, Bombay for waiving their usual regulations and inviting and permitting me to submit this thesis for the Acharya Degree and for permission th publish it here.

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B. Chaitanya Deva

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For those who may not be familiar with Indian nomenclature of notes the following will be of help :

Symbol	Indian name	Sol-fa name
Sa	Sadja	Do
re	Kōmal rishabh or Suddha rishabham	Re flat
Re	Suddha Rishabh or Catussruti rishabham	Re
ga	Kōmal gāndhār or Sādhārana Gāndhāram	Mi flat
Ga	Suddha Gāndhār or Antara Gāndhāram	Mi
Ma	Suddha Madhyama	Fa
ma	Tīvra Madhyama	Fa sharp
Pa	Pancama	Sol
dha	Kōmal dhaivat or Suddha dhaivatam	La flat
Dha	Suddha dhaivat or Catussruti dhaivatam	La
ni	Kōmal nisād or Kaisiki nisādam	Si flat
Ni	Suddha nisād or Kākali nisādam	Si

In the case of Indian names, the first is according to Hindustani system and the second according to the Karnatak system. In the text, a dot under the symbol indicates the lowering by a register and a dot above raising by a register.

The following abbreviations are used in the bibliography :

BDCRI : Bulletin of the Deccan College Postgraduate Research Institute, Poona.

JASA : Journal of the Acoustical Society of America, New York.

The Tonal Structure of the Tambura

Abstract

The present thesis is a report on the preliminary investigations carried out by me on the tonal structure of the tambura.

The investigation was first started as a part of the researches in the psychophysics of speech and music being conducted under the guidance of **Prof. C. R. Sankaran** at the Phonetics Laboratory, Deccan College, Poona.

The point of departure for the study of the psychophysics of our music was the drone – its function and significance. Its psychology and historical influence on the dynamics of the fluctuations of our musical forms were already discussed by the present writer (Psychology of the Drone in Melodic Music, **BDCRI. Vol. 10,** 1950 and the Emergence of the Drone in Indian Music, " **Jl. Madras Music Academy,** 1952). The tonal structure of the tambura which is the main drone in our music was the subject of the next step in the research programme and the present thesis is a report of this work.

The thesis is planned as follows :

Chapter I –

The psychology of the drone and its importance in our music is briefly discussed and the importance of the tonal background due to the drone is emphasised. This naturally leads to the problem of consonance. A critical review, therefore, of the various theories of consonance (**Helmholtz, Stumpf, Seashore**) is made.

Chapter II.

The experiment conducted by me and the results thereof are reported. The harmonic analysis of each string of the tambura (for two types of tuning–Pa and Ma) with and without **juari** thread is reported in detail. To study the nature of the string-bridge contact of the instrument, photographs of this point were taken when the string was actually vibrating. From observation and the photographs it was seen that the peculiar mode of vibration and the tonal structure of the tambura string was not only due to the grazing contact of the string with the bridge (as reported by Sir **C. V. Raman**) but due to a periodic damping of the vibration of the string at the bridge. The string was actually seen to leave the bridge and return to it periodically.

This introduces a periodic damping with the attendant consequences on the mode of the vibration of the string and hence its tonal quality. The main inferences are (1) that the damping and its consequences are in the nature of impulses, (2) such an impulse occurs once in a cycle, (3) the consequences are : production of harmonics which are not to be expected according to **Young-Helmholtz** law, production of inharmonics and subharmonics, emission of tones as impulses, and periodic change in the frequency of the string, (4) with the removal of the **juari** thread there is a very perceptible difference in tone-quality attributable to the lack of

periodic damping, (5) with the removal of the **juari** thread there is an increase in the frequency of the string, obviously owing to the decrease in its length.

A tonal system as that of the string may be treated as analogous to a bar – the frequencies of the fundamental and harmonics being analogous to distances along the bar from one end and the intensities of the tones to masses of points along the bar. Then, a frequency, termed the tone-centroid, may be calculated analogous to the centre of gravity of a rod. If C is the tone centroid, n the frequency of a tone and c the amplitude of the tone, then,

$$C = \frac{\Sigma nc^2}{\Sigma c^2}$$ (c^2 being proportional to the intensity of the tone)

for any given tonal spectrum.

The tone-centroid (and correspondingly, the intensity centroid $I = \frac{\Sigma nc^2}{\Sigma n}$ are indicative of the energy shifts. These have been calculated for each string (with and without **juari** thread.)

The tone-centroid is also indicative of the quality of the instrument and hence can be used to study the relation between the structure and quality of musical instruments and voices.

Chapter III

As already pointed out, the mode of vibration of tambura-string is such that there is every reason to expect the presence of subharmonics and inharmonics. The detailed calculation of these is, indeed very complicated and laborious - which may involve operational calculus and other such mathematical tools. Though such labour would give an exhaustive tonal spectrum, it was not worth taking, considering the accuracy of the records taken (for the recording could not be made in an acoustically conditioned room which, however, is now being constructed in the Laboratory.)

Hence an easier method due to **Scripture** (vide, his **Researches in Experimental Phonetics**, Carnegie Institution of Washington, 1906) was adopted. The results are only approximate and are presented in this chapter.

Chapter IV

It is a well known fact that when a tone is sufficiently intense a person hears not only the tone present outside his ear, but other tones which are harmonics of the external tone. These harmonics, as they are produced within the ear, are termed as aural harmonics. When two such tones impinge on the ear, then not only the aural harmonics but also tones, called combination tones, are heard. Since these tones are not present outside the observer, they are called subjective tones. (This is a convenient term, inspite of psychologists like Max F. **Meyer**).

The frequencies of aural harmonics are obviously integral multiples of the frequency of external tone. Combination of tones -- summation and difference tones - may be represented by the general formula $(mh \pm nl)$ where h and l are the higher and lower primaries and m and n are integers.

The most commonly accepted theory to explain the production of these subjective tones is that the transmission mechanism of the ear responds non--linearly beyond certain values of the intensity of the stimulating tone (i. e. **Hooke's** law fails to hold good at these levels). Mathematically this state is expressed as

$$X = f(p) = a_0 + a_1 p + a_2(p)^2 + a_3(p)^3 + \dots\dots$$

where X = displacement of the cochlear fluid near the oval window of the cochlea.

p = Pressure variation in front of the ear drum

a_0, a_1, a_2, a_3 ...... constants characteristic of the non-linearity of the auditory mechanism.

p is proportional to the amplitude of the external tone; the constants ao, al, etc., may be calculated from the characteristic curve of the ear. Substituting these values in the expansion of the above equation wherein the terms for the frequencies of the tones are involved, it is possible to calculate the number, frequencies and amplitudes of subjective tones.

This has been done for both **Ma**- and **Pa**-types of tuning.

Chapter V

Summary of the experiment and results therefrom are given.

The relation between the nature of tambura--string vibration and the alpha phoneme theory is discussed.

The probable avenues for further research are pointed out.

Chapter I

Introduction

The psychology of melodic perception and integration is one of the basic factors in all music; for, without melody - a tonal progression in time - there can be no music. All other factors are of secondary importance in comparison with melody. The only true and basic fact in music is the extension of tone (itself a temporal phenomenon) in time. Harmony is the progression of groups of tones in time. Rhythm is the factor that decides the nature of extension of a tone; and, in the final analysis, rhythm and tone (both being forms of motion) can not be separated. The simplest form of melody is the progression of one tone at a time - the usual melodic music. When groups of tones (chords) progress in time - we have harmonic music with all its beauties of organum and counterpoint.

The nature of melodic valuation at any period in the history of music is conditioned and determined by two backgrounds - the 'remote' and the 'immediate'. The remote background consists of the musical heritage and training of the musician and the immediate one consists of the drone.[1]

The function of the drone is exceedingly important in melodic music. A detailed study of this has been presented elsewhere,[2] but a brief review will not be out of place.

1. The integration of tones into a melody is indeed a very complex process and as yet we do not have sufficient data to go upon for the formation of a well-knit theory of melodic integration. Prof. **E. B. Newman** of the Psychology Laboratory, Harvard, says:[3]

'There are several inconsistencies in our knowledge of this field. Some people who seem to be tone- deaf may have bad pitch discrimination. But it's not nearly bad enough to explain their failure to recognize and identify tones. One of the men in our laboratory, for example, could never identify a telephone signal which we used which consisted of three notes in succession. He simply said that the notes didn't mean anything to him. On the other hand this man obviously understands speech with great facility. His pitch threshold may be two or three times larger than normal but he is nothing in the world but truly tone-deaf.

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1. C. R. SANKARAN and B. Chaitanya **Deva**, Postulational Methods and Indian Musicology, **Jl. Bom. Univ.**, 18. pt. 2. 73 ff (1949)
2. B. Chaitanya **Deva**, Psychology of the Drone in Melodic Music, **BDCRI** 9 (1950)
3. In a personal communication to the present author, d. 25. 12. 1949. A very good treatment of the problem of tonality may be found in G. REVESZ's recent book, **Introduction to the Psychology of Music** 55-112 (Longmans, 1953). See also my review of this book in **BDCRI** 14. 154-60 (1955).

'There seem to me to be at least two other components in the problem which one must disentangle. One is the question of memory. Evidently people vary widely in the richness of their memories of a given type of sensory impression. Many problems of music depend clearly upon the facility and extent of such tonal memory. In addition, there is an underlying problem of the perceptual organization of melodic material. A few very preliminary experiments done by Dr. G. A. **Miller** in this Laboratory suggest the likelihood that that a series of notes will hang together in a melody depends clearly upon such factors as 1. rate, 2. tonal distance, 3. direction of progression, and probably on others which we do not yet know about.

'I suspect that the problem of tonality which you raise probably can't be answered fully until we understand something about both the perceptual organization of tones and the nature of learning in this area.

'....these are very off-hand comments and they don't represent any substantial knowledge which we have in this interesting field.'

Apart from these considerations is the important fact that each tone, each group of tones, each relation of tones and groups are related to one psycho-acoustic level - the tonic (which, when given a definite acoustic character, is the drone). Here is the importance of the tonic and the drone. It is necessary to emphasise that it is the **relationship** that is important and not the actual acoustical value of a tone. Further it must be pointed out that such a relationship of relationships is more fundamentally a psychological one and not one of respiratory convenience.[4] In all this organization the tonic forms the base psychologically and when this gets an acoustic form, it is termed the drone.

Secondly, the tonic (or the drone) represents a state of tonus of the organism. An emotional state tends to make the organism tense and the organism always will tend to return to a state of rest or tonus. Parallel to this is the tonal movement. Any departure from the tonic engenders a state of tension which is resolved by returning to the tonic. Such departures–their extent and nature - may again be used as factors of integration, comparable to tonal distances from the drone (or tonic).

In harmonic music we find the same phenomenon of starting and finishing a musical piece on the same key. This method of musical composition, it is said, is being departed from in the twelve-tone (atonic) music of Arnold **Schoenberg** and his followers. This would be introducing new kinds of states of tension without resolving them acoustically. But, perhaps, the mind does find a rest after the music is over - a kind of post-tonic. This is a different problem, not immediately relevant to the present work.[5]

4. W. HOSKINS, on General Semantics and Music, **General Semantics Bulletin** Nos. 6, 7, 49 ff.

5. An interesting study of such problems has been made, from the point of view of depth psychology in A. EHRENZWEIGs **The Psychoanalysis of Artistic Vision and Hearing** (Routledge Kegan Paul, 1953).

Also, these psychological functions of the drone seem to have exerted an extraordinary force on the development of our musical forms. Obviously, if the drone is so important in melodic integration, a musician cannot afford to shift his drone. It must be steady, **acala**. But yet, prior to about 14th – 15th century, we do find a musical system with a variable **Sa (cyuta Sa)** and a variable **Pa (cyuta Pa)**. But after **Ramamatya** this type of musical nomenclature disappears and the beginnings of the present system appear.[6]

Thirdly, and this aspect is the subject of this thesis, the drone provides a tonal background for a musician. Against this background each tone of the melody finds a relationship. Such a relationship may be pleasant, unpleasant, neutral or any other shade of affective tone. Such affective values are employed as the material for emotional tones of the melodies.

It is often objected that an intonation governed by the drone will not be accurate and be one of harmonic progressions and the notes of the melody will tend to fuse with the harmonics of drone-stringts. This is, indeed, a mistaken view. For one of the finest musical pleasures is to place a note which is not of simple harmonic relation to the drone-tones and to perceive the curious ' dissonance '.

The nature of consonance and dissonance is not yet very clearly understood. Till we have a better knowledge of auditory mechanism - both peripheral and central-we cannot develop a unified theory of consonance. The most widely accepted theory is due to **Helmholtz** who sought to explain consonance in terms of beats. Beats may occur between the objective as well as subjective tones. Of this **Seashore** says :--

'In the main this is correct and applies in large part to all scales; but there are important exceptions, so that no such simple rule can be applied throughout to determine the degree of consonance or dissonance.'[7]

The four factors that are usually said to determine consonance-dissonance are :-[8]

1. Smoothness - roughness: is a feeling of tonal relation explainable on the operation of beats.
2. Purity-richness: this scale corresponds fairly to the order of consance-dissonance. There is an order of increasing richness from unison to minor second which can be determined by harmonic analysis.

   Seashore distinguishes not only the degree of purity but also the kind of purity.
3. Based on these two physical facts is the sense of blending or the sense of belonging together.

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6. I have discussed this problem (which, as far as I am aware, has not been studied previously by anyone in a systematic manner) in my, Emergence of the Drone in Indian Music, **Jl. Music. Acad.**, Madras, 23 (1952).
7. Carl E. SEASHORE, **Psychology of Music** 126 (McGraw-Hill, 1938).
8. **ibid.**, 129–33.

4. Another factor advanced by **Stumpf** is the ability to hear as one tone (fusion) two tones played simultaneously. But this factor is so much governed by the reaction time of the subject that it is difficult to accept it as a criterion of consonance-dissonance ranking.

The ranking of diads, taking these factors into consideration is given in Fig. I (after **Malmberg**).

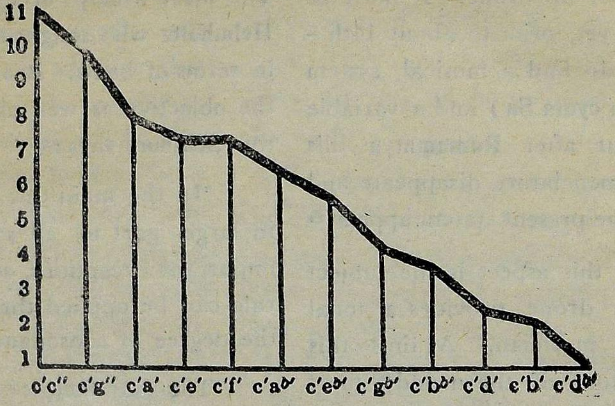

Fig. I. Order of merit of interval in the consonance - dissonance series, according to MALMBERG. (By permission from, **Psychology of Music**, by Carl E. SEASHORE. p. 132. Copyright 1938, M. Graw-Hill Book Company, Inc.)

The problem of consonance has been subject to studies from various points of view of which the following are the most important :-[9]

I. Theories of correspondences:

I have chosen to so term those theories of the ancient Hindus, Greeks, Chinese, and etc., who sought to understand consonances as expressible by the correspondence of physical phenomena to certain numerical ratios. It was not merely the fact that consonance is (often, though not always) representable by simple numerical ratios - e. g. 1 : 2, 2 : 3, 3 : 4, 4 : 5, etc. but, such ratios and numerical relations governed all **'phenomena'** of which music was a vital form. So that musical relationship was the same as stellar relationship, for instance.

These theories have much to recommend them, specially that they seek to express the basic character of all **'phenomena'**. But, such a mode of thought and communication is possible only when the social atmosphere is more Ideational than Sensate and hence its lack of appeal to the modern 'scientific' musicologists, to whom Sensate values are truer.

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9. G. REVESZ, **ibid.**, Ch. 6 (Part II) 77-92 A. DANIELOU, **Introduction to the Study of Musical Scales** (London, India Society, 1934) P. A. SOROKIN, **Social and Cultural Dynamics**, Vol. 1. Ch. 12 (Allen & Unwin, 1937).

II. Physical theories :

To this category belong those theories which seek to understand consonance only on the basis of physical facts and do not let metaphysical facts 'disturb' them. Of these the most common are the theories of (1) **Helmholtz** and (2) **Stumpf**.

(1) **Helmholtz** tried to explain consonance-dissonance by referring to the beats between two (or more) tones. The greater the prominence of beats, the less consonant the tones concerned. This to a great extent appears to bear out in practice. Octave (1 : 2) fifth (2 : 3) sixth (3 : 5) etc., show almost no beats or very few beats. But Ri (8 : 9) or ri (15 : 16) show prominent beats, and hence are not as consonant as the previous intervals.

It is usually taken that beats are most prominently perceived when they are about 33/sec., and when more than 132/sec., they are not perceivable. But considering the following diads :

Ṣa – Ri,	Sa – Ri,	Ṡa – Ṙi,	S̈a – R̈i	– Notes
120 – 135	240 – 270	480 – 540	960 - 1080	– Frequencies
15	30	60	120	– Beats.

The number of beats change as octave level changes. That is, the **same interval** has **different degrees of consonance** ! Similarly between **Sa** : **Ri** (240 : 540) there are 300 beats – beyond the limits of beat-perception, hence they must not sound dissonant.

Another fact responsible for consonance, according to **Helmholtz**, was the coincidence of the overtones of the primary tones. Those primaries with greater number of coincident overtones are more consonant and **vice versa.** However, if two pure tones are sounded they should be neutral, as they have no partials.

These are some of the serious deficiencies in **Helmholtz's** theory.

Further refinements of these conceptions were due to **Preyer** and **Kreuger** who took into consideration the difference tones.

Though the factors of roughness (due to beats) and coincidence of overtones do affect consonance of tonal relationship, they are not sufficient to explain fully the nature of consonance.

(2) Since consonance cannot be fully explained in terms of partials of tones, **Stumpf** sought for the basis of consonance in the nature of the primaries themselves. As a result of his experimental investigations he put forward the theory of fusion. The main point in his theory was that when two tones were sounded together the greater they were heard as one tone (fused in to one) the greater their consonant relationship.

However, later investigators showed that there could be tones that were very consonant with each other, but yet could be easily be recognized and separated from each other. Similarly, there were diads that could with great difficulty be separated but yet were dissonant.

Revesz rightly points out that we must distinguish between consonance and concordance. The former is a relationship of two tones and the latter that of more than two tones. Two tones, say **Ga & Dha** may sound concordant, but on the addition ol say **ma** may sound discordant.

I would further suggest here that the place of **tonal memory** and **tonal retro-relation** must be carefully studied in relationship to this problem. The question is, "What is the relation of two or more tones in terms of consonance, when they occur one after another ?". Obviously, tonal memory and tonal retro-relation (and postrelation) are involved. This is of primary importance in melodic music.

However we approach the problem, it is necessary to know the tonal background provided by the drone. And it is towards such an understanding that this investigation is directed.

Chapter II

Elctro-Acousic Analysis of Tambura

As a part of the studies in the psychology of melodic music (the point of departure being the psychophysics of the drone)[1] the following preliminary investigations of the tonal characteristics and the related physical phenomena of the tambura were undertaken.

The **tambura** is the main drone instrument in Indian music (though seldom used to accompany wind instruments like the **shanai** and **nagaswaram**). Its tonal structure and the mode of vibration of its strings are very characteristic.[2] No other musical instrument–not even the violin- is so rich in its timbre; its development and use in our music has been one of the finest fruits of the genius of our instrument makers.

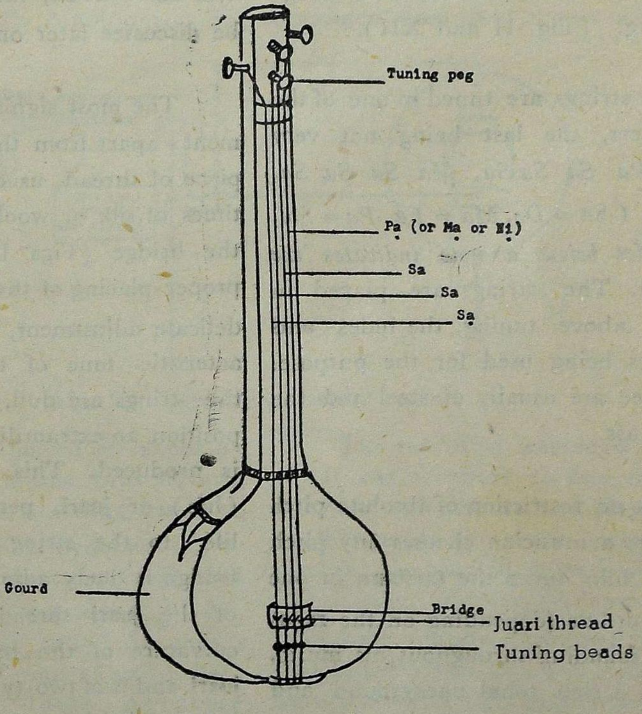


Fig. II

The tambura

The instrument (Fig. II) consists of a resonator, pear shaped and made of wood or of a gourd, which is covered by a curved wooden board and at its upper end joined to a very long, hollow neck. At the distal end of the neck are four tuningpegs – two on the top and one on either side. On the

board of the resonator rests the typical wide and curved bridge. It is usually made of ivory, buck-horn or of certain kinds of wood (most common being of **sissoo - Dalbergia sissoo**). It is held on to the resonator by two small protruberances on the board which fit into small depressions in the legs of the bridge; the high tension of the strings also help to keep the bridge well pressed on to the board. This enables the removal of the bridge whenever necessary for cleaning and filing. Near the lower end of the resonator and on it are four beads - one for each string-for finer tuning. (Fig. II and XII).

The four strings are tuned in one of the following orders, the last being not very common :— *Pa Sa Sa Sa, Ma Sa Sa Sa, Ni Sa Sa Sa* (*Sa = Do, Ma = Fa, Pa = Sol, Ni = Si; the dot below a note indicates the lower register*). The strings are played in order of the above tuning, the index and middle fingers being used for the purpose. The first three are usually of steel and the last one of brass.

There is no restriction of absolute pitch in this tuning; a musician chooses any pitch convenient to him, tunes the **tambura** in one of the above orders (depending on the **rāga**) and keeps it sounding throughout his music. This provides a rich tonal background and reference for correct intonation.

The actual method of tuning used by me is first to tune the middle two strings to the required pitch; this is done in such a way that there are no beats between the two. Next the Sa string is tuned in such a way that on playing it, the middle Sa strings resonate. The Pa string is tuned in such a way that it resonates on plucking Sa string; if this string is to be tuned to Ma, then the correctness of tuning is shown when Sa strings resonate on playing Ma string. Tuning the string to Ni is more a matter of auditory judgement - it is tested so that it is a perfect fifth of the Ga (Mi) produced from Sa string. (It will be noted that Pa = 3/2 Sa and Sa = 3/2 Ma; the significance of this will be discussee later on).

The most significant part of the instrument - apart from the sloping bridge - is the piece of thread, usually of cotton but sometimes of silk or wool, under each string on the bridge (Figs. II and XII). It is the proper placing of the thread, which is a very delicate adjustment, that generates the characteristic tone of the strings. Without it the strings are dull, but with the thread in position an extraordinary retinue of overtones is produced. This thread is called **jeevan** (life) or **juari**, perhaps because it 'gives life' to the string. The curvature of the bridge is finely adjusted to suit the thickness of the **juari** thread; very commonly this curvature of the bridge is also called the **juari** and is of two types: **gol**, when the sound is 'rounded' but persists for a long time and **chiran** when the tone is brighter but dies down quicker.

The approximate dimensions of the **tambura** used in the present experiment are:

Table I

The circumference of the gourd (at its greatest) = 4' 2"

Height of the gourd = 14"

Total height of the instrument = 4' 6"

Vibrating length of the strings ≐ 38"

String	Mean Diamater	Dimensions of strings Lengths a*	(Approx.) b*	in cms c*
Pa	.53 mm	96.8	97.7	98.4
Sa Sa	.51 mm	96.9	97.6	98.5
Sa	.85 mm	96.9	97.7	98.5

* See further discussion.

Experiment

The electro-–acoustic recording was carried out in a room of about 20'X20'X18' which was not acoustically conditioned. The experimental set up is shown below (Fig.III).

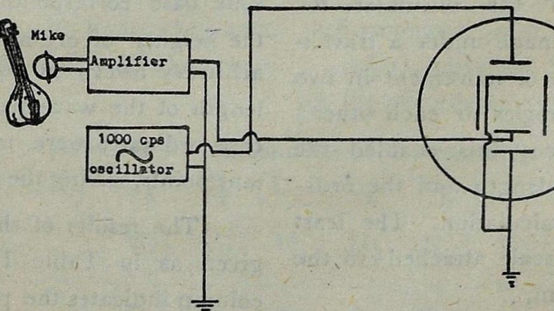
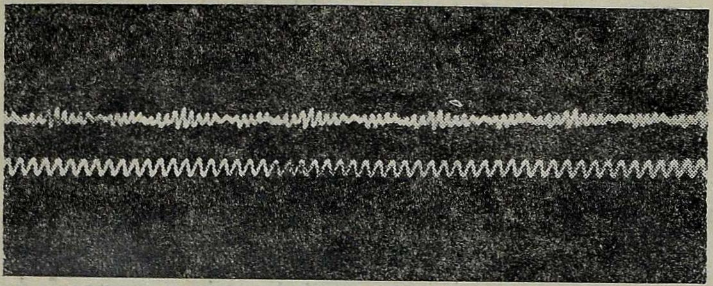


Fig. III The experimental set up

The **tambura** was played at about 8" from the microphone: it was held vertical, on the floor, the player squatting beside it. The microphone was of Shure dynamic velocity type. The response of the microphone was led through an amplifier (constructed by Sri. **P. C. Ganeshsundaram** of this Laboratory) and fed to one of the beams of an oscillos-cope (Cossor–Industrial 1049). A battery maintained oscillator of 1000 cps, which served as the time-base, was fed to the other beam of the oscilloscope. The spots (of the beams) were kept steady and the film was run horizontally at the rate of 3"/sec. The camera used was of Avimo manufacture.

The recording was done in two stages: (1) It was necessary to find the cumulative effect of all the strings, played one after another as in the order given above. The camera was started after playing the last (Sa) string. By this process all the strings were kept vibrating together (but, in diffe-rent phases) and the oscillographs obtained were records of the cumulative effect of the four strings. Such records were taken of the two types of tuning Pa Sa Sa Sa and Ma Sa Sa Sa. The records and other data of this part are referred to as Pa (all) or Ma (all) in the text. (2) The record for each string was taken separately. In each case records

were made of the tone of the string with and without **juari**; the string was in no way disturbed, but only the thread was slid out and removed. Thus the difference in tone due to the **juari** could be studied. Obviously, the Pa string required retuning during the experiment.



Sample oscillogram of Sa string (Upper track.). Lower track:– Time base of 1000 cps.

Measurements of the ordinates for Fourier analysis were made under a travelling microscope, capable of movement in two directions (at right angles to each other) in the horizontal plane; this enabled the measurement of wave length and the ordinates to be used for calculation. The least count of the vernier scale attached to the microscope was .001 cm.

Runge's method was followed for the Fourier analysis of the records.[3] The method consits in measuring the length of a repetitive pattern*, the number of cycles in the time base corresponding to this length and the heights of ordinates (from any chosen arbitrary level) at equal intervls along the length of the wave pattern. In the present case ordinates were measured at 12 equidistant points, giving the data for six harmonics.

The results of these measurements are given as in Table II. The first vertical column indicates the phase angle, the second the numerical order of the ordinate designated as U and the succeeding columns the values of measured heights in cms. The data from actual measurements are given in Appendix I.

Table II

Ordinate Measurements

Designation of the string :— Zero reading : (Arbitrary)

		Values of heights for four periods					
θ	U	1	2	3	4	Mean	Difference from zero reading. (ordinates)‡
0	0	...	...	...	...	...	......
30	1	...	...	...	...	...	......
60	2	...	...	...	...	...	......

* The wave patterns are not perfectly identical, but may be considered so, to a first approximation, for the purposes of Fourier analysis.

‡ The mean of four values is subtracted from an arbitrary value (zero reading) to get the actual value of the ordinate.

Table II (*Contd.*)

θ	U	1	2	3	4	Mean	Difference from zero reading. (Ordinates)
90	3	...	...	...	...	...	......
120	4	...	...	...	...	...	......
150	5	...	...	...	...	...	......
180	6	...	...	...	...	...	......
210	7	...	...	...	...	...	......
240	8	...	...	...	...	...	......
270	9	...	...	...	...	...	......
300	10	...	...	...	...	...	......
330	11	...	...	...	...	...	......

The general theorem of Fourier for the expansion of an arbitrary function of a variable x is given as :

$F(x) = a_0 + a_1\cos x + a_2\cos 2x + a_3\cos 3x + \ldots\ldots$
$+ b_1\sin x + b_2\sin 2x + b_3\sin 3x + \ldots\ldots$

where $a_0, a_1, a_2, \ldots\ldots b_1, b_2, b_3$ are constants. U being a known function of x, the above may be re-written as :

$U = a_0 + a_1\cos x + a_2\cos 2x + \ldots a_6\cos 6x$
$+ b_1\sin x + b_2\sin 2x + \ldots b_5\sin 5x$ for 12 equidistant ordinates.

The values of the constants a and b are computed from the values of U_0, U_1, $\ldots\ldots U_{11}$, obtained from the measurements of the records as follows :

Table for the computation of the constants.

	U_0	U_1	U_2	U_3	U_4	U_5	
		U_{11}	U_{10}	U_9	U_8	U_7	U_6
Differences :		w_1	w_2	w_3	w_4	w_5	
Sums :	v_0	v_1	v_2	v_3	v_4	v_5	v_6

	w_1	w_2	w_3
	w_5	w_4	
Sums :	r_1	r_2	r_3
Differences :	s_1	s_2	
	r_1	q_0	
	r_3	q_2	
Differences :	t_1	t_2	

	v_0	v_1	v_2	v_3
	v_6	v_5	v_4	
Sums :	p_0	p_1	p_2	p_3
Differences :	q_0	q_1	q_2	
	p_0	p_1		
	p_2	p_3		
Sums :	l_0	l_1		

(Values in these columns are multiplied by the sine values against them)

	Sine terms			Cosine terms			
Sine 30	r_1			q_2	$-p_2$ p_1		
Sine 60	r_2	s_1 s_2		q_1			
Sine 90	r_3		t_1	q_0	p_0 $-p_3$	t_2	l_0 l_1
Sums of 1st column	...	...		...	...		......
Sums of 2nd column	...	...		...	...		......
Sums	$..=6b_1$	$..=6b_2$	$..=6b_3$	$..=6a_1$	$..=6a_2$	$..=6a_3$	$..=12a_0$
Differences	$..=6b_5$	$..=6b_4$		$..=6a_5$	$..=6a_4$		$..=12a_6$

For actual calculaations see Appendix I.

The amplitdes of the various harmonics (cl = fundamental, c_2 = second harmonic, etc.,) are calculated from the equation:[4]

$$c_1 = \sqrt{a_1^2 + b_1^2}\ ; \quad c_2 = \sqrt{a_2^2 + b_2^2}\ ; \quad c_3 = \sqrt{a_3^2 + b_3^2} \text{ and so on.}$$

Results

Tables III and IV and Figs. IV to IX summarise the results of the analysis of the wave forms of the various tones.

Table III

Note	Frequency (cps)	No. of Harmonic	Amplitude (c) in cms.	c^2	C_n/C_1	Tone Centroid (C) in cps	Intensity Centroid (I) as c^2
Sa String with juari :							
Sa	72	1	.01034	.0001069	1		
Sa	144	2	.01249	.0001561	1.208		
Pa	216	3	.01104	.0001218	1.067	360.6	.001051
Sa	288	4	.03498	.0012240	3.384		
Ga	360	5	.01979	.0003040	1.914		
Pa	432	6	.04900	.0024010	4.740		

(Contd.)

Table III (*Contd.*)

Note	Frequency (cps)	No. of Harmonic	Amplitude (c) in cms.	c^2	Cn/C_1	Tone Centroid (C) in cps	Intensity Centroid(1) as c
Sa String without Juari :							
Sa	72	1	·01281	·0001641	1		
Sa	144	2	·01792	·0003210	1.399		
Pa	216	3	·004718	·00002225	0·3683		
Sa	288	4	·09506	·009036	7·422	307·3	·003684
Ga	360	5	·03972	·001577	3.101		
Pa	432	6	·03842	·001476	2.999		
Ma String with juari :							
Ma	96	1	·03311	·001096	1		
Ma	192	2	·004336	·00001880	0.134		
Sa	288	3	·02824	·0007973	0.8529	411.4	·0021
Ma	384	4	·02851	·0008128	0.8612		
Dha	480	5	·06215	·003457	1.877		
Sa	576	6	·04467	·001995	1.349		
Ma String without juari :							
Ma	96	1	·02627	·0006903	1		
Ma	192	2	·02385	·0005611	0·908		
Sa	288	3	·003601	·000013	0·1371	384.6	·0007838
Ma	384	4	·01775	·0003152	0.6959		
Dha	480	5	·04274	·001826	1.627		
Sa	576	6	·02650	·0007021	1.009		
Pa String with juari:							
Pa	108	1	.01287	.0001656	1		
Pa	216	2	.01004	.0001009	0.6582		
Ri	324	3	.007196	.00005178	0.3117	402	.0001496
Pa	432	4	.01226	.0001503	0.9069		
Ni	540	5	.01536	.0002356	1.4220		
Ri	648	6	.01183	.0001400	0.8453		
Pa String without juari :							
Pa	108	1	.006789	.00004609	1		
Pa	216	2	.007271	.00008377	1.071		
Ri	324	3	.02522	.0006357	3.715	422.5	.001511
Pa	432	4	.08447	.007133	12.45		
Ni	540	5	.01429	.0002014	2.095		
Ri	648	6	.003667	.00001345	0.5043		

Table III (*Contd.*)

Note	Frequency (cps)	No. of Harmonic	Amplitude (c) in cms.	c^2	C_n/C_1	Tone Centroid (C) in cps	Intensity Centroid (I) as c
Sa_1 String with juari :							
Sa	144	1	.01032	.0001066	1		
Sa	288	2	.02598	.0006751	2.522		
Pa	432	3	.03301	.001089	3.203	485.8	.0004404
Sa	576	4	.01364	.000186	1.323		
Ga	720	5	.01821	.0003314	1.767		
Pa	864	6	.01883	.0003546	1.828		
Sa_2 String with juari :							
Sa	144	1	.01477	.0002181	1		
Sa	288	2	.01545	.0002387	1.046		
Pa	432	3	.03181	.001012	2.153	451.5	.0003027
Sa	576	4	.01587	.0002518	1.075		
Ga	720	5	.01493	.0002229	1.011		
Pa	864	6	.009167	.00008403	0.6205		
Sa_2 string without juari :							
Sa	144	1	.01111	.0001234	1		
Sa	288	2	.01411	.0001991	1.27		
Pa	432	3	.02570	.0006608	2.313	652.4	.001091
Sa	576	4	.01139	.0001296	1.025		
Ga	720	5	.06197	.0038410	5.578		
Pa	864	6	.01080	.0001166	0.9721		

Note : For convenience in calculation and comparison the lowest frequency (**Sa**) has been taken as 72 cps (both with and without juari), though this is not the exact value.

The signs 1 and 2 attached to **Sa** have no musical connotation but only indicate the first and second **Sa** string.

Table IV

Pa-tuning

Note	Freq.	$c^2 \times n^2$	Energy ratio to Sa=1	Tone centroid	Intensity centroid
Sa	72	.5619	.05597		
Pa	108	1.932	.1924		
Sa	144	10.04	1.000		
Pa	216	10.77	1.073		
Sa	288	177.3	17.66		

(Contd.)

Table IV (*Contd.*)

Note	Freq.	$c^2 \times n^2$	Energy ratio to Sa=1	Tone centroid	Intensity centroid
Ri	324	5.436	.5414		
Ga	360	50.89	5.069	416.7	.0001891
Pa	432	868.2	86.47		
Ni	540	68.70	6.843		
Sa	576	145.5	14.49		
Ri	648	58.88	5.865		
Ga	720	287.7	28.66		
Pa	864	327.8	32.65		
Ma-tuning					
Sa	72	.5619	.05597		
Ma	96	10.21	10.17		
Sa	144	10.04	1.000		
Ma	192	.6700	.06673		
Pa	216	5.709	.5686		
Sa	288	244.3	24.33		
Ga	360	50.89	5.069	420.6	.001512
Ma	384	120.2	11.97		
Pa	432	840.2	83.69		
Dha	480	796.5	79.33		
Sa	576	808.6	80.54		
Ga	720	287.7	28.66		
Pa	864	327.8	32.65		

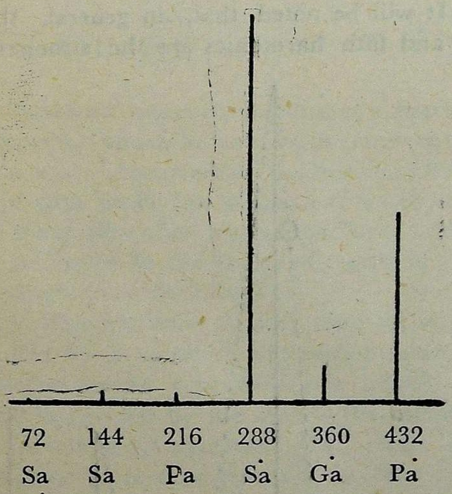


Fig. IV Pa-tuning — all strings

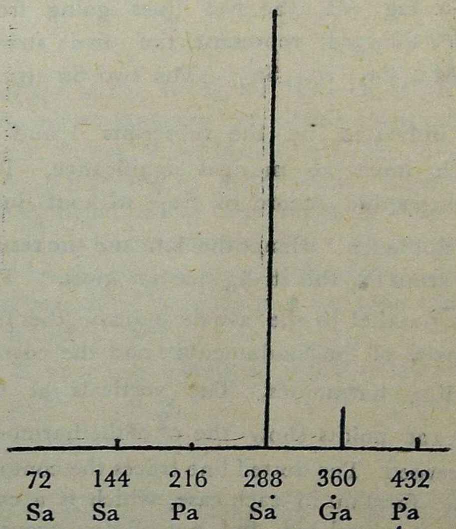


Fig. V Ma-tuning — all strings

The primary interest of the investigation, as it was started, was in the energies of the various tones produced by each string, as the intensities (and hence the loudness) of these tones are the first concern musically. Since c^2 is proportional to intensity, for any given frequency, it is this that is tabulated and shown in the figures.

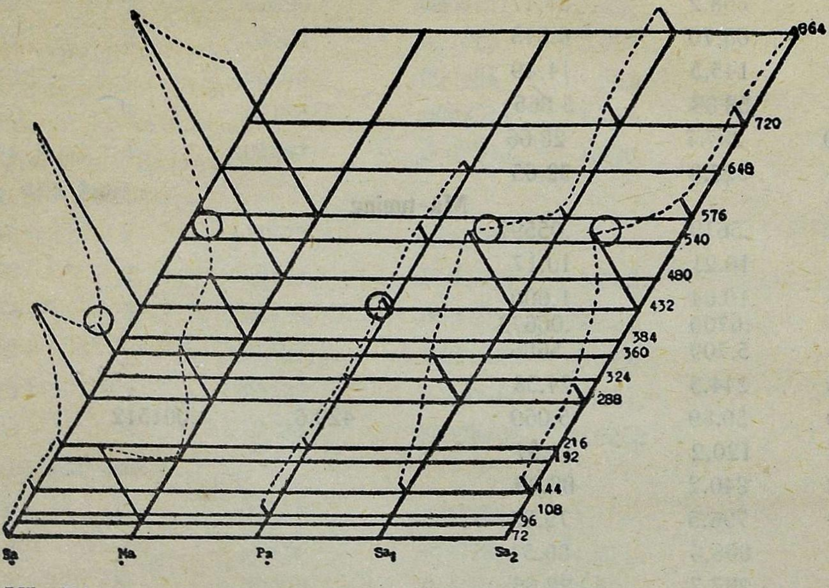


Fig. VI Diagram showing the harmonic structure of each string with **Juari.**

In Fig. VI, the five lines going from reader forward represent the five strings Sa, Ma, Pa, Sa_1, Sa_2 (The two Sa strings are indicated by the subscripts 1 and 2, which have no musical significance. The oscillographic record of Sa_1 without juari was not taken. Hence the data and the results therefrom for this string are not given). The lines parallel to the x-axis indicate the frequencies of the fundamentals and the corresponding harmonics. The verticals at the relevant points show the c^2 of the harmonic concerned. The dotted line traces the contour of the formant in each case, which is a convenient visual aid and not of any acoustic significance.

It will be noted that, in general, the third and fifth harmonics are the strongest.

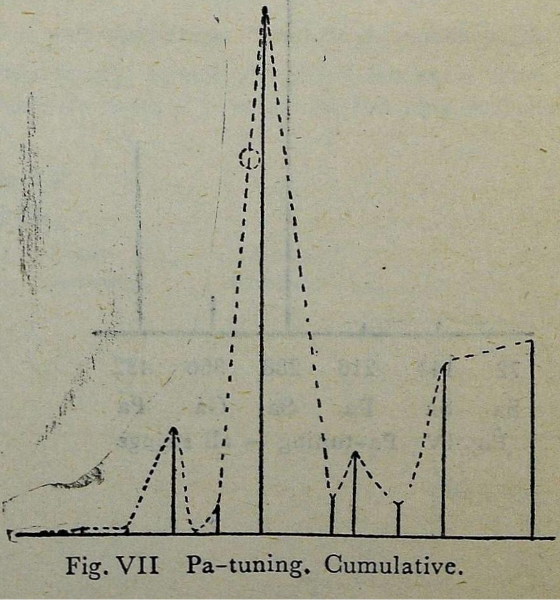


Fig. VII Pa-tuning. Cumulative.

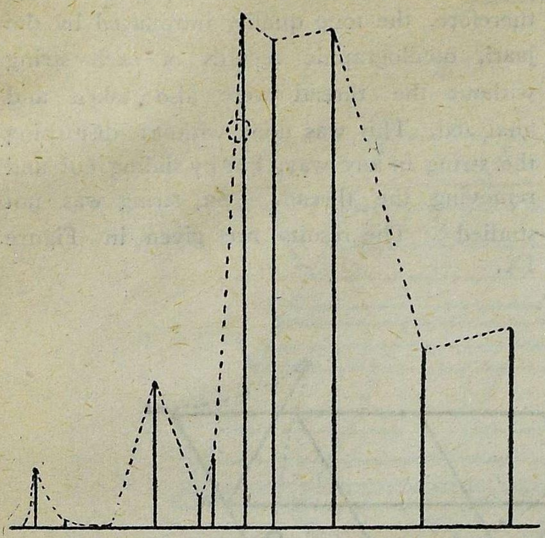


Fig. VIII Ma-tuning. Cumulative.

Figs. VII and VIII show the combined c^2 of each frequency as produced from the various strings, the data for this tonal spectrum being given in Table IV. As a musician has the middle two strings tuned to his tonic (Sa), the intensity of this tone in relation to the others in the tonal spectrum is of significance. Therefore, the ratio of the energy ($n^2 c^2$)* of each of the frequency to the energy of Sa (produced by three strings) has been computed aud indicated as the height of the line at each frequency in the tonal spectra. In the order decreasing energy, the various frequencies for each type of tuning are arranged thus :

Pa-tuning : Ṗa, P̈a, Ga, Ṡa, S̈a, Ṅi, R̈i, Ġa, Pa, Ga, Ṙi, Pạ, Sạ.

Ma-tuning : Ṗa, S̈a, Ḋha, P̈a, G̈a, Ṡa, Ṁa, Ma, Ġa, Sa, Pa, Ma, Sạ.

Fletcher has given the following qualitative equation as a characterizable definition of a formant[5] : S = Characteristic of the formant, f = frequency of fundamental or harmonic, I = respective intensity :

$$S \longrightarrow \frac{f_2/f_1}{I_2/I_1}, \frac{f_3/f_1}{I_3/I_1}, \frac{f_4/f_1}{I_4/I_1}, \ldots\ldots \frac{f_s/f_1}{I_k/I_1}$$

Abbott[6] however, found that a particular frequency, which he terms as the tone centroid, is a more characteristic quantitive definition and with profit can be used as an indicator of the quality of an instrument. Tone centroid is analogous to the centre of gravity of a bar. It is obtained as follows :

The frequency of each tone (n) is multiplied by its intensity and such products for the frequencies concerned are added; this total is divided by the sum of the intensities of all the tones relevant. The result is a frequency, the tone centroid, which is characteristic of the string. If the sum of all the 'frequency × intensity' of all the tones due to all the strings of an instrument be divided by total intensity of all the tones, then the tone cøntroid of the instrument is obtained and this is characteristic of the instrument.

In the present investigation c^2 has been used instead of intensity and the tone centroid is defined as:

$$C = \frac{\Sigma\ (nc^2)}{\Sigma\ c^2}.$$

Similarly the intensity centroid, I is

$$I = \frac{\Sigma\ (nc^2)}{\Sigma\ n}$$

* Strictly the energy in any sound is given by mn^2c^2, m being the mass of the vibrating body. But, as the mass of the string and 'air particles' may be assumed to be constant, the above quantity is directly proportional to the energy of the sound.

The tone centroids of each string and for the cumulative spectra and indicated as dotted circles on the contour in the figures.

The tone centroids of each string and for the cumulative spectra are indicated as dotted circles on the contours in the figures.

It has been said, already, that it is the proper placing of **juari** thread that gives the **tambura** its characteristic timbre. To study, therefore, the tone quality introduced by the **juari**, oscillographic records of each string without the thread were also taken and analyzed. This was done without disturbing the string in any way, but by sliding out and removing the thread. (**Sa**$_1$ string was not studied). The results are given in Figure IX.

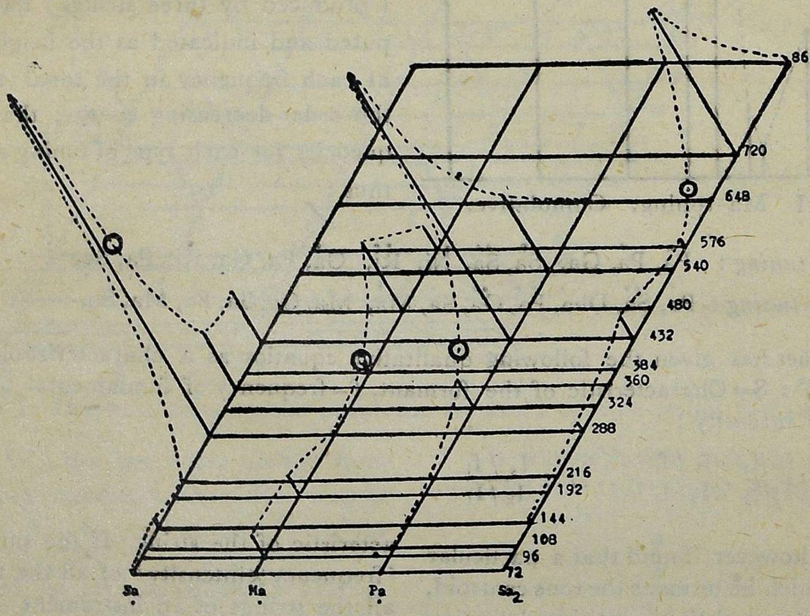


Fig. IX Diagram showing the harmonic structure of each string without **Juari**.

The difference due to the introduction and removal of the **juari** may be quantitatively studied either by making use of the changes in energies or the changes in the amplitudes of the harmonic. It is obvious that the actual energy or the amplitude of the harmonic is not significant in this respect, for these depend upon the force of plucking of the string. What is, therefore, of significance is the relation of each partial to its fundamental. The relation of the c^2s may be easily made out from the figures. The relation of the amplitudes are given in Table III as Cn/C_1; these values are shown in Figs. X and XI.

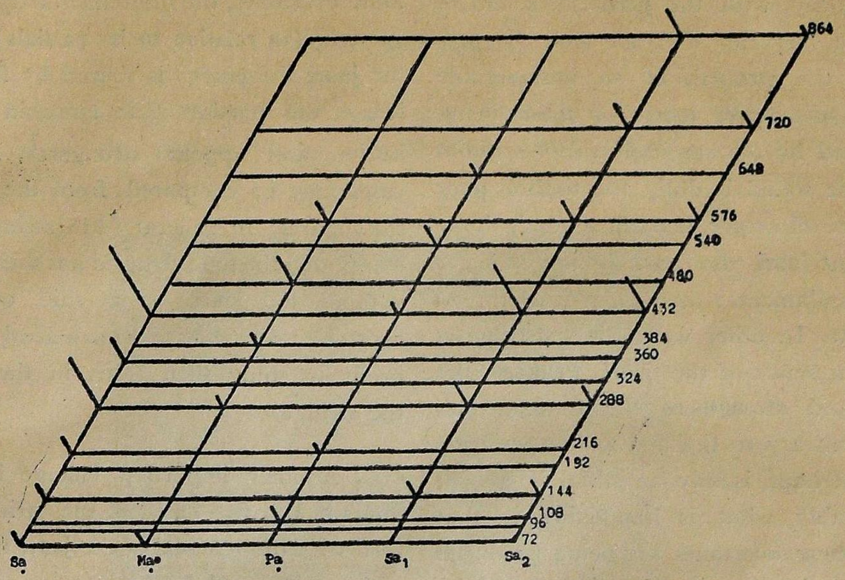


Fig. X. Diagram showing the ratios of the amplitudes of the harmonics to that he fundamental. (With **Juari**.)

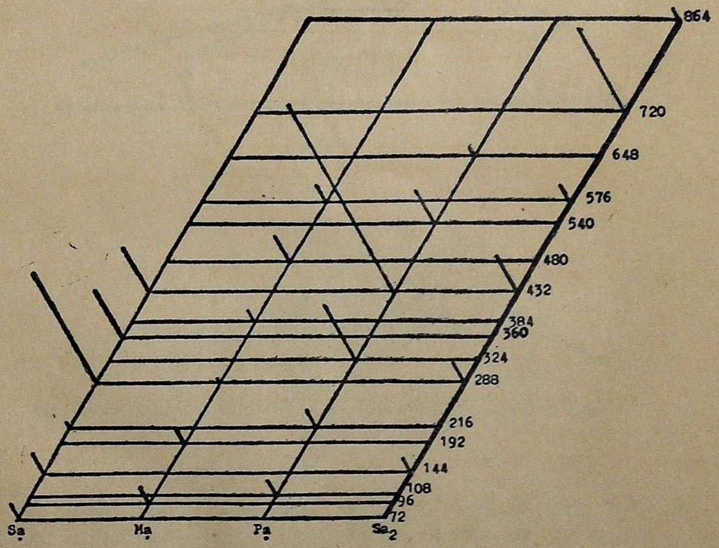


Fig. XI. Relations of the amplitudes of the harmonics to that of the fundamental for each string. (Without **Juari**).

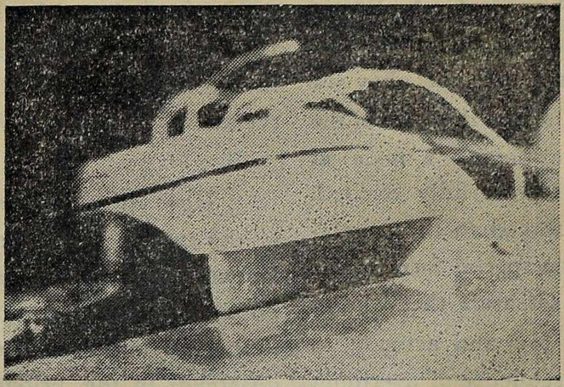
Auditorily, with the **juari** the sound of the string is very rich and one feels the presence and the strength of an innumerable overtones (of course, many of these being tones created by the ear itself). But without the **juari** the sound is dull, the timbre poor in overtones. However, acoustic analysis shows that without **juari** the partials are stronger than the fundamental than when **juari** thread was present. In other words, it first appears that the presence of the **juari** **weakens** the overtones and **strengthens** the fundamental, which is contrary to the auditory experience. But, this, perhaps, is only an initial inference; more probably what is happening is that with **juari** more overtones are being produced than without it; the energy being shared by more overtones, the fundamental appears to be stronger **(in relation to its partials)**. Without the **juari** the energy is shared by fewer tones, hence, the partials gain more in the distribution and appear stronger.[7] Both the cases may be compared from the tables and the figures; in general (Ma string being an exception) the tone centroid has shifted **upwards** without the **juari**. This shift of the tone centroid may be used conveniently and with profit to study such shifts in the energy of the string.

Another important change is that the fundamental frequency of the string is increased with the removal of the **juari** thread. This will be discussed later.

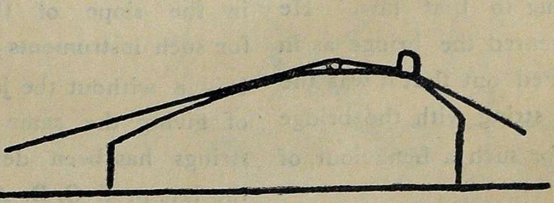
[Continued]

The Bridge.

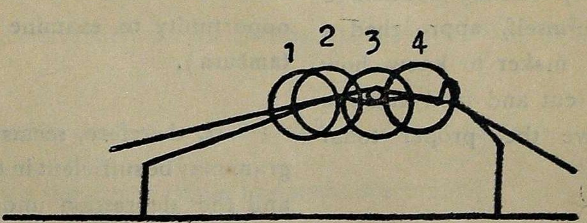
The characteristic tone and the complicated mode of vibration of the strings of the **tambura** are definitely attributable to the structure of the bridge of the instrument. (Fig. XII).



(a)



(b)



(c)

Fig. XII. (a) Close up photograph of the bridge **in situ**, with the **juari** thread and the tuning beads. (b) Diagram of the bridge following RAMAN. (c) Diagram as per my observation. The various parts photographed (see Fig. XIV) are shown as numbered circular regions. (See however text).

The approximate dimensions of the bridge, made of buck-horn in the instrument studied here, have already been given. The **juari** thread was of cotton No. 40.

The first study of this instrument (and the vina) was made by Sir **C. V. Raman** about three decades ago[8]. He observed that the strings of the **tambura** did not obey the **Young-Helmholtz** law–i. e., in the case of the **tambura**-string the tone having a node at the point of plucking is powerfully elicited. He says, 'We are thus forced to the conclusion that the effect of the special form of the bridge is completely to set aside the validity of the **Young-Helmholtz** law and actually to manufacture a powerful sequence of overtones including those which ought not to have been elicited according to that law.' He diagramatically represented the bridge as in Fig. XII b and pointed out that it was the grazing contact of the string with the bridge that was responsible for such a behaviour of the string.

The present writer, wanting to examine the phenomenon for himself, approached a very good instrument maker to know how exactly the bridge was cut and filed and the slope adjusted to give the proper tonal quality to the strings.

The actual making of the bridge revealed a detail which had missed **Raman**'s notice. He has given the slope of the surface of the bridge as fairly uniform (Fig. XII b). But closer examination, during the making, showed that the maker slightly smoothens out, with sand-paper, the small portion of the bridge where the thread is to be kept. This smoothening creates a small depression, hardly noticeable[9]. Apart from adjusting the curvature of the bridge to suit the thickness of the **juari** thread[10], the depression divides the bridge into two humps (parts 2 and 4 in Fig. XIIc) with the depression between them.

How far such a depression is essential in the surface of the bridge is not certainly known. Because, firstly, **sitar, been and vina** do not have such a thread though (according to the instrument maker) there is no essential difference in the bridges of the **tambura** and of the these instruments (the difference lying in the slope of the bridge, being less for such instruments); secondly, a **tambura** bridge without the **juari** thread but capable of giving the same quality of tone to the strings has been developed (it is said, by the late prof. **G. B. Achrekar**, Poona, though many folk drones do have such bridges without the thread. I have not yet had the opportunity to examine such a bridge on a **tambura**).

It, therefore, seems that **Raman**'s diagram may be sufficient in essentials. The thread and the depression under it seem to serve the function of making the contact between the string and the bridge **very slight at the** first hump (part 2 in Fig. XII c). That this contact should be extremely light will be evident from what follows.

For purposes of observing and photographing the vibration of the string at the bridge, the following arrangement was set up. Thg **tambura** bridge with the strings on was illuminated from one side by lamp of 100 c. p., the rays of which were passed through a convex lens and vertical slit, before they fell on the string. This concentrated beam of light produced a very clear silhouette of the bridge and the string when viewed from the opposite side. The contact of the string and the bridge was viewed through an ordinary laboratory microscope, which was fixed to the resonator of the instrument by means of plasticine, so that it could easily be shifted when necessary. After personal observation, photographs were taken of these silhouttes with a Robot camera of only the Sa string (Fig. XIII).

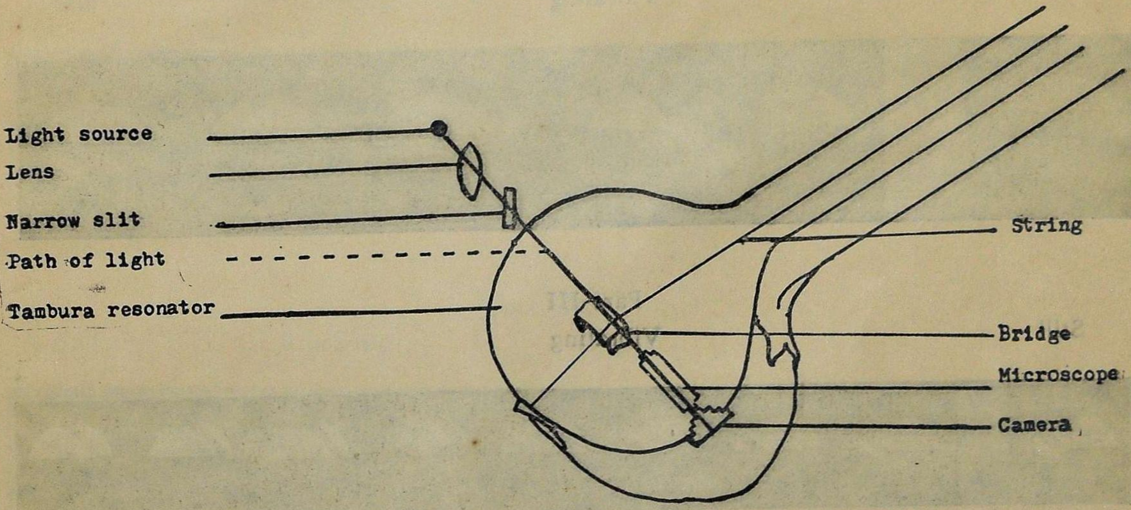


Fig. XIII. Arrangement for photographing the string-bridge contact.

The bridge-string contact was photographed at four parts:

(1) Just when the string began to touch the bridge.

(2) Just after this portion, at the first ' hump. '

(3) At the **juari** thread.

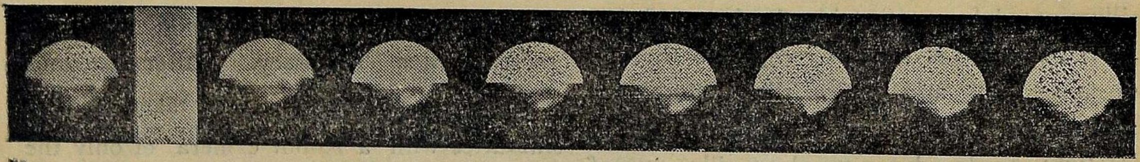
(4) After the thread, at the second 'hump' (See Figs. XIIc & XIV)

Fig. XIV. Photographs of string-bridge contact

Part I

Still

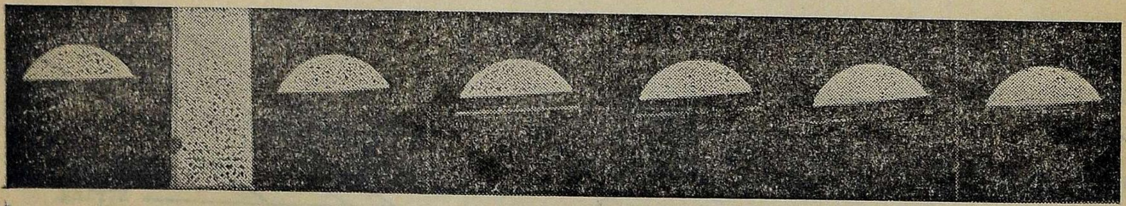
Vibrating



Part II

Still

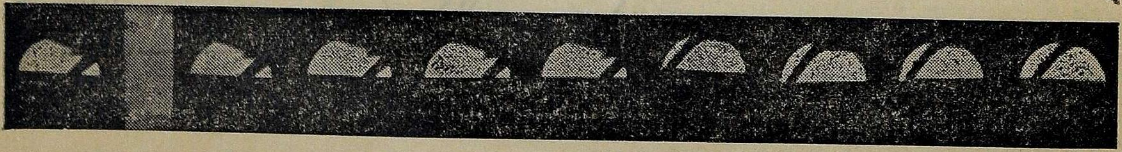
Vibrating



Part III

Still

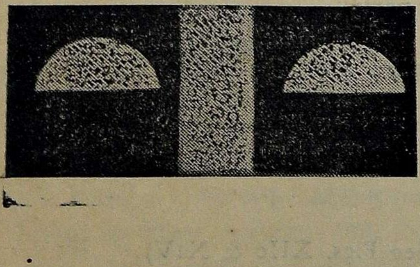
Vibrating



Part IV

Still

Vibrating

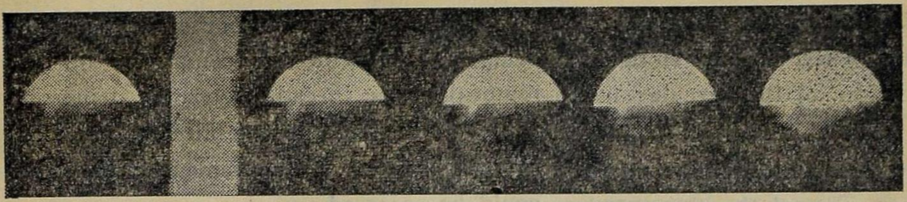


With Juari

Fig. XIV cont.

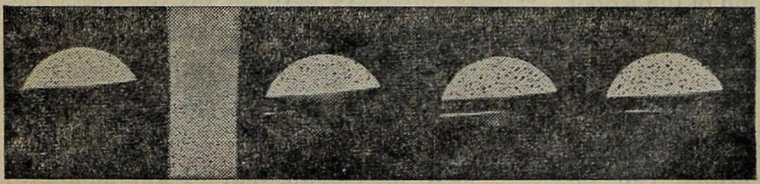
Part I

Still Vibrating



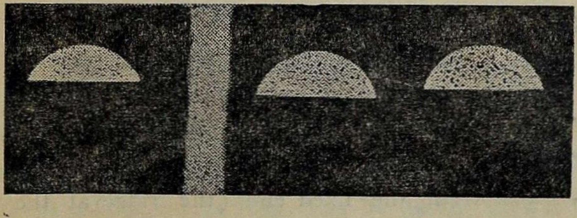
Part II

Still Vibrating

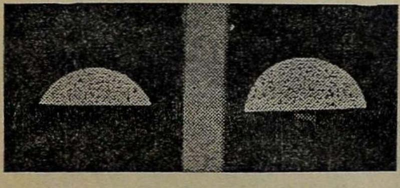


Part III

Still Vibrating



Still Vibrating Part IV



Without Juari

In the left first of Fig. XIV are the photographs of the bridge-string contacts when the **juari** thread was in place and at the second when without the thread. In each row, the first picture is that of the string at rest and following ones when it was vibrating. In the lower portion of the circular field of view can be seen the silouhette of the bride and above this the silouhette of the string as a thick bar. Though the photographs have been juxtaposed to show the various stages, they are **not** of successive instants during vibration. (The operation of the Robot Camera precludes any such photography of very closely successive instants).

Part 1. (a) In the 'still' the grazing contact of the string with bridge can be clearly seen. When the string is vibrating, the gap between the bridge and the string widens appreciably.

(b) Without the thread the widening of the gap is not to such an extent as in (a).

This will be obvious from the following.

Part 2. (a) **With the thread, the string was seen to leave the bridge completely, periodically.** Most probably this happens once in a cycle, though this could not be confirmed by my observation due to the high frequency of the string. A detailed study of this is now going on in our laboratory.

(b) **Without the thread, the string does not leave the bridge at all.**

Part 3. (a) The string can be seen lifted up from the bridge due to the thread; again the intermittent contact of the string and the bridge during vibration can be observed.

(b) The gap between the string and the bridge is obliterated is completely on the removal of the thread. Even the depression caused by the smoothening (already described) can not be seen. No movement of the string could be observed.

Part 4. (a) and (b). There was no observable difference between the two cases, the string being in close contact with the bridge. The string can hardly be distinguished from the bridge in the photographs.

It is unfortunate that I am unable to compare these photographs with those of **Raman,** as he has not published them. The following tentative suggestion may be made on these preliminary observations of mine.

It will be noted that with the removal of the thread the vibrating length of the string is decreased slightly. With the thread, the vibrating length of the string is measurable from the fixed end (the ridge at the top of the **tambura**) to the thread (l_0) or to the contact behind the thread ($l_0 + d$), d being the distance between the thread and either contact, before and after it. But with the removal of the thread, the string contacts the bridge at Part 1 (and never loses this contact, which is, **even then, not a point contact**) and hence the vibrating length of the string is decreased from l_0 (or $l_0 + d$) to $l_0 - d$. These lengths are shown as $a = (l_0 - d)$,

b=(l_0) and c=(l_0 +d) in Table I. Assuming that the tension of the string is not varied appreciably by the introduction or the removal of the thread, it is obvious that the fundamental frequency of the string must be increased (due to decreased length) on removing the **juari** thread. Analysis of the oscillographic records does show such an increase in frequency (with the exception of Ma string). The data are summarised here:

Fundamental Frequencies of

	Sa	Ma	Pa	Sa_1	Sa_2
With the thread:	72.73	96.36	105.3	140.04	140.04
Without the thread:	74.08	94.06	108.1*	--	145.4

Though there is a close corespondence between the frequencies observed and calculated from the lengths of each string, it did not have any predictable accuracy. A more careful measurement and electro-acoustic analysis may reveal a more definable relationship. The crucial point in the investigation will then be the determination of the permanent node and the temporary nodes at the bridge.

A very important point in the mode of the vibration of the string is the fact that with the thread, the string leaves the bridge (at part 2), and hence is damped periodically at this part. It may also be noted that it is not the grazing contact alone that is significant (for, even without the thread the contact of the string with the bridge is a grazing one), but that the string suffers a periodic damping at the string-bridge contact and the reaction of the bridge at this part is in the nature of an impulse. Though **Raman** has not noted this fact, perhaps this is what he implies by, 'The forces exerted by the string on the bridge at this point (the point of grazing contact – Note that **Raman** does not mention anywhere in his paper that he has observed the string leaving the bridge) are probably in the nature of impulses occuring once in each cycle of vibration of the string[11]. It may also be probable that a temporary node is created in the string at part 2 which shifts to part 3 or 4 on the lifting of the string from the bridge. The effect of this would be to introduce phase changes during vibration and also a longitudinal transition of the nodes on the string. **Raman** has already shown that even in ordinary strings there is a vibratory movement of node longitudinally[12]. Our preliminary observations have revealed such a movement of nodes in the **tambura** strings; more detailed observation may show how far these movements are different from those

(*Since the Pa string was retuned during the experiment, this may not be taken as providing unequivocal evidence. No oscillogram of Sa_1 without the **juari** was taken).

in ordinary strings and to what extent they are due to the structure of the bridge.

The observations made and reported in this thesis are in the nature of a **preliminary** investigation into the very complicated, but nevertheless entrancing, tonal structure and mode of vibration of the **tambura** strings which deserve detailed investigation. The ollowing are some of the problems, raised by this investigation, which may yield rich fruit :

1. It was reported that oscillograms of all the strings vibrating together were taken. However, the fundamental of these records was Ṣa; hence a harmonic analysis would not show the presence of Ṃa and P̣a, as they are inharmonics of Ṣa. Hence they were not analysed to great detail, but the spectra are given for the sake of completeness. (Figs IV and V).

In the section on tuning it was stated that P̣a string vibrates (when correctly tuned) on plucking Ṣa. But these tones are related as P̣a : Ṣa = 3:2. Such a resonance would mean that Ṣa string (and all other strings as well, for this can be easily tested) is producing an inharmonic. (I have often observed that the string produces resonance even in Ṃa string). Therefore, Fourier analysis is inadequate and does not give a true picture of the formant. A mathematical tool like Vercelli analysis or Bessell function may be more adequate.

2. Discussions in our laboratory on the present investigation suggested that subharmonics might be produced from the strings. This was tested for; on plucking Sa, Ṡa string resonated. (Sa: Ṣa = 2:1)

These two problems require careful confirmation and setting up of controlled experiments. The following solutions of this problem may be suggested :

(i) The strings may really be producing inharmonics, in which case a more suitable form of mathematical analysis other than Fourier analysis will have to be used. (It is important to note that these are inharmonics only in the physical sense; psychologically they are very 'harmonic').

(ii) The resonance observed may not be due to inharmonics at all (though the mode of vibration of the string indicates every possibility of the production of inharmonics) but due to the second harmonic of the resonating string; for instance Ṗa string may not be resonating to a tone of $1\frac{1}{2} \times$ Ṣa (an in-

harmonic) but to the third harmonic of Sa (Pa). In such a case Pa is being forced to oscillate in two loops by Pa produced from Sa string. A similar explanation is applicable to the observed subharmonics.

(iii) It can be shown[13] that due to phase differences introduced by the intermittent contact of the string with the bridge, the string may be taken to be 'vibrating' with twice its length. Then, the inharmonics become the harmonics of this doubled length.

Obviously, delicately controlled experiments are necessary to test the validity of these suggestions or any other possible explanations.

3. It can be observed that, on plucking the string, overtones are not heard as continuous tones but as impulsive and intermittent tones. The easiest to observe is the 5th harmonic of Sa. It is suggested by **P. C. Ganeshasundaram**[13] that Operational Calculus may serve well in the study of the impulsive character of the tones and their production.

4. These diverse facts observed – the change in node, periodic damping, and impulsive production of tones – would lead to periodic changes in frequency, amplitude and formant of each string. Herein lies a vast field for the study and mathematical definition of the sonance of the **tambura.**[14] The mode of growth of each tone and its harmonic, and the transition from transient to regular vibration of the string would be analogous and relatable to portomento in singing and speech; this may be another approach to correlation of the phenomena of speech and music, with the developing of a general expression for both in the wake of **C. R. Sankaran's** Alpha-phoneme theory.

5. The study of the tone centroid of musical instruments and voices (in speech and singing) is another venue for intensive research. Firstly, the tone centroid is indicative of the distribution of energy. Secondly, as shown by **Abbott,** the tone centroid is a very good index of the quality of instruments (and I may add, of voices); this fact can be used to investigate the relation of structural and tonal characteristics of our musical instruments.

* * *

Addendum:

To gain further information regarding the mode of vibration of the **tambura** strings, photographs (of exploratory nature) were taken at a convenient point of the string. The arrangement was similar to the one shown in Fig. XIII. For purposes of comparison similar photographs of ordinary strings were also taken. The results are shown in Fig. XIVa.

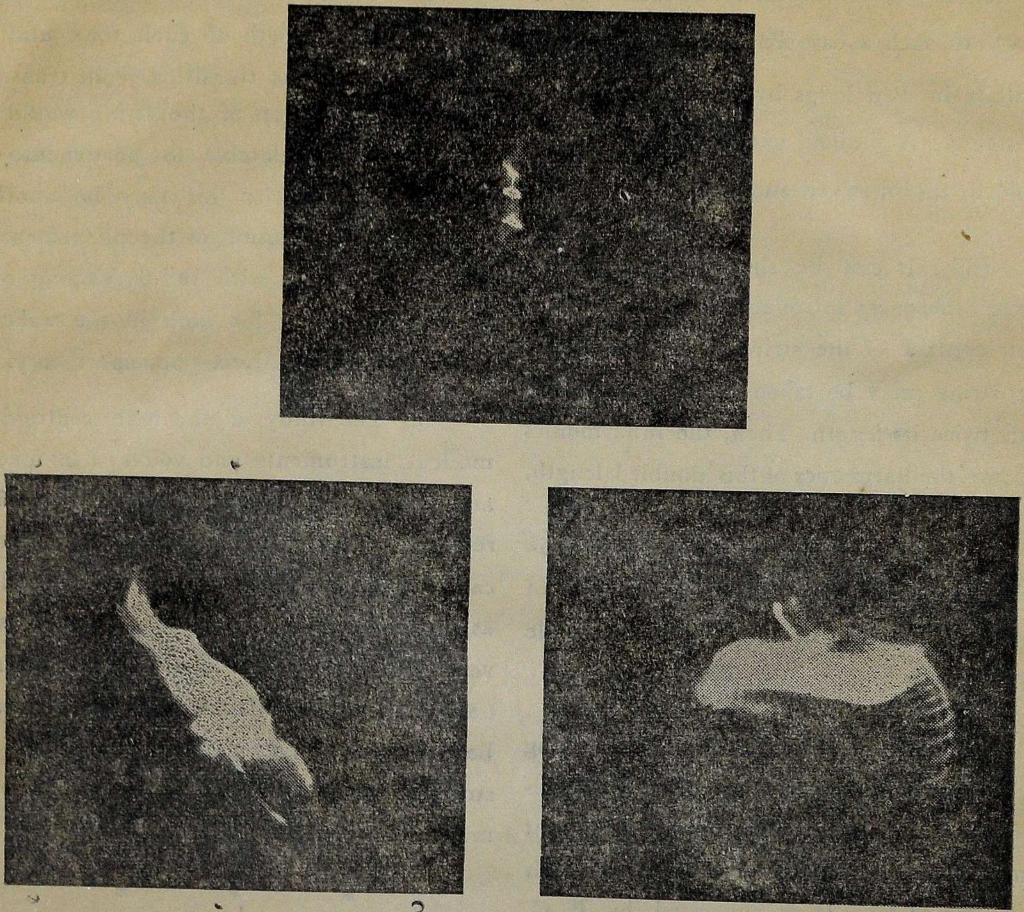


Fig. XIVa. Photographs of vibrating strings. The upper one is of an ordinary sonometer string with changing phase and tension. Of the lower ones, on the left is of the **Tambura**-string with **juari** and on the right without **juari**. The spiral nature of vibration in all cases can be seen clearly.

The upper photograph is that of an ordinary string. During vibration the tension of the string was being changed gradually, introducing phase changes. The lower photographs are those of the tambura string, with and without juari. In both cases it can be seen that the strings are vibrating both in the vertical and horizontal plane. When actually observed the illuminated point is seen to trace a circular path (or other figures depending upon the constancy of tension). In the case of the tambura, however, the horizontal component seems as prominent as the vertical. This is in fair comformity with the hypothesis suggested by **Ganeshasundaram**[13].

Notes and References:

1. B. Chaitanya DEVA, Psychology of the Drone in Melodic Music, **BDCRI** 9 (1950).

2. The only other experimental study of the **tambura** (and **vina**) has been that of C. V. RAMAN, On Some Indian Stringed Instruments, **Proc. Ind. Ass. Cult. Sc.**, 7 (1921–22) 29 ff; wherein he has suggested the probable mechanism for the peculiar behaviour of the strings of this instrument. (See infra).

3. G. A. CARSE and G. SHEARER, **A Course in Fourier Analysis and Periodogram Analysis for the Mathematical Laboratory**, Edinborough Math. Tracts., No. 4. 17 ff (1915).

4. E. W. SCRIPTURE, **Researches in Experimental Phonetics**, Carnegie Inst. Washington Publ., No. 44 (1906).

5. H. FLETCHER, Loudness, Pitch and Timbre of Musical Tones and their Relation to the Intensity, Frequency and the Overtone Structure, **JASA** 6. 60 (1935).

6. R. B. ABOTT, Response Measurement and the Harmonic Analysis of Violin Tones, **JASA** 7.113 (1936).

7. These facts may be compared with the results of the acoustic analysis of violin tones by P. H. EDWARDS, A Method of the Quantitative Analysis of Musical Tones, **Phys. Review,** (1911) p. 37:

"1. The lowest tones of the violin are almost lacking in fundamental.

2. Muting increases fundamental and decreases overtones in the lower tones...; in the higher ones it decreases all the constituents."

I must here point out that often it is calculated that the harmonics are weaker than the fundamental progressively. Cf., L. R. PATWARDHAN, **Tamburaci pahili tar,** (Marathi), **Sangit Kala Vihar** Jan; 1954, p. 14 ff. Such calculations are obviously based on HELMHOLTZ's estimations for **ideal** strings. They have no correspondence in actuality which is almost the reverse.

8. C. V. RAMAN, **ibid.**

9. The depression has been enlarged purposely in Fig. XII c for clarity.

10. The process is a delicate one. If a thread of any other thickness than that initially used by the maker be employed as **juari,** the tone is different.

11. C. V. RAMAN, **ibid.**

12. C. V. RAMAN, The Small Motion at the Nodes of a Vibrating String, **Researches in Acoustics** (Baptist Mission Press, Calcutta, 1914).

13. P. C. GANESHSUNDARAM, A Note on the Electroacoustical Analysis of the Tambura, (to be published).

14. Sonance is the "successive changes and fusions which take place within a tone from moment to moment" and is defined as "that aspect of tone quality which results from fluctuations in pitch, intensity, time and timbre within a tone". Carl E. SEASHORE, **Psychology of Music** 103, 108 (Mc Graw-Hill, 1938).

Appendix I

Harmonic Analysis (Sample)

In the following pages of this Appendix are given the details of the data and the calculations for obtaining the amplitudes and frequencies of the first six harmonics. The first part is the table of measurements and the second the calculations by RUNGE'S method.

The first vertical column, θ, gives the phase angle, the second the number of ordinate, U; the four columns which follow give the numerical values measured along the y-axis for four wave periods when the vibrations are fairly steady. The next column is the mean of the four.

The last column gives the mean ordinate. This is obtained by subtracting the mean measured height in the fourth coulumn, from an arbitrary zero reading. These mean ordinates are used for further calculations.

For each period the number of time-curve correspondence, n, is also given. This is the number of waves of the time-curve (1000 cps) which correspond to one period of the tone curve. Obviously, the fundamental frequency of the tone is 1000/n cps.

Pa-tuning:- all strings. Zero reading : 10.5 cm.

		Height in cm.					
θ	U	1	2	3	4	Mean	Ordinate in cm.
	n :	14	13.5	14	13.5	13.75	
0	0	10.406	10.388	10.359	10.350	10.376	0.124
30	1	10.210	10.248	10.213	10.200	10.218	0.282
60	2	10.333	10.330	10.322	10.315	10.325	0.175
90	3	10.150	10.207	10.175	10.187	10.175	0.325
120	4	10.341	10.337	10.337	10.314	10.332	0.168
150	5	10.229	10.246	10.218	10.202	10.224	0.276
180	6	10.309	10.334	10.303	10.305	10.313	0.187
210	7	10.234	10.229	10.228	10.233	10.231	0.269
240	8	10.295	10.297	10.282	10.268	10.286	0.214
270	9	10.254	10.227	10.216	10.236	10.233	0.267
300	10	10.325	10.298	10.292	10.300	10.304	0.196
330	11	10.218	10.228	10.200	10.184	10.208	0.292

Fundamental frequency : 72.73 cps.

It may be noted for those who are not acquainted with this method that 12 ordinate measurements are necessary by obtaining the amplitudes of 6 harmonics.

APPENDIX I

Calculations :

U.	.124	.282	.175	.325	.168	.276	
		.292	.196	.267	.214	.269	.187
Diff.		—.010	—.021	+.058	—.046	+.007	
Sum	+.124	+.574	+.371	+.592	+.382	+.545	+.187

w.	—.010	—.021	+.058
	+.007	—.046	
Sum	—.003	—.067	+.058
Dif.	—.017	+.025	
r, q.	—.003	—.069	
	+.058	—.009	
Diff.	—.061	—.059	

v.	+.124	+.574	+.371	+.592
	+.187	+.545	+.382	
Sum	+.187	+.149	+.753	+.592
Dif.	—.063	+.029	—.009	
p.	+.311	+1.149		
	+.753	+.592		
Sum	+1.064	+1.741		

	Sin terms			Cos terms			
Sin 30 = .5	-.0015			-.0045	-.3765 +.5745		
Sin 60 = .866	-.05802	-.01472 +.02165		+.02511			
Sin 90 = 1	+.058		-.061	-.063	+.3110 -.5920	-.054	+1.064 +1.741
Sum of 1st col	+.0565	—.01472		—.0675	—.0654		+1.064
Sum of 2nd col	—.05802	+.02165		+.02511	—.0175		+1.741
Sum	—.00152	+.00693	—.061	—.04239	—.0829	-.054	+2.805
Diff.	+.11452	—.03637		—.09261	—.0479		—0.677

$a_0 = +.234$ $a_1 = -.00707$ $a_2 = -.0138$ $a_3 = -.009$ $a_4 = -.08$ $a_5 = -.01544$

$b_0 = -.056$ $b_1 = -.00025$ $b_2 = +.00116$ $b_3 = -.01$ $b_4 = -.00606$ $b_5 = +.01909$

Amplitudes of first six harmonics :

Note	Frequency	Amplitude (c) in cms.	Note	Frequency	Amplitude (c) in cms.
Sa	72*	.007074 - c_1	Ṡa	284	.08024 - c_4
Sa	144	.01385 - c_2	Ġa	360	.02456 - c_5
Pa	216	.01346 - c_3	Ṗa	432	.05600 - c_6

* Though the more accurate value for the fundamental is 72.73 cps, the more simple number 72 cps has been taken for further calculations, for the sake of convenience.

Chapter III

Inharmonic Analysis

From the foregone discussion there is every reason to expect the production of not only harmonics not to be looked for according to **Young-Helmholtz** law, but also of inharmonics and subharmonics.

In Chapter II in the section on the tuning-process it was said that on plucking the Sa string Pa string also vibrated. Similarly on the plucking of Ma string Sa string was seen to resonate. (I have sometimes noticed – though not always – that even Ma string resonates on plucking Sa). Now, the following relations hold for the above tone-pairs :-

Sa : Pa = 2 : 3 (Pa = $1\frac{1}{2}$ × Sa)
Ma : Sa = 2 : 3 (Sa = $1\frac{1}{2}$ × Ma)
Sa : Ma = 3 : 4 (Ma = $1\frac{1}{3}$ × Sa)

During the discussions on the present work in our laboratory it was suggested (by **Dr. P. C. Ganeshsundaram**) that quite probably even sub-harmonics were being produced. Auditory judgment could not be really decisive owing to the presence of subjective-tones. However, on the plucking of Sa string Sa string vibrated (Sa = $\frac{1}{2}$ × Sa).

Now it is possible that :

(1) The resonance of such inharmonically or sub-harmonically tuned strings may not be directly due to an inharmonic produced from the plucked string, but due to an octave of the inharmonic. For instance, Pa may not be resonating due to an actual tone Pa = $1\frac{1}{2}$× Sa being produced from Sa, but due to Pa = 3 × Sa. In the latter case Pa-string is being forced to vibrate in two loops by Pa produced from Sa.

Again Sa may not be vibrating due to a Sa = $\frac{1}{2}$ Sa from Sa string, but directly by Sa itself (Sa = 2 Sa) in two loops.

(2). The inharmonics and sub-harmonics, if produced, may be due to the resonator of the instrument.

(3). Inharmonics and sub-harmonics are actually produced. Though the processes suggested in (1) and (2) may be occuring, the mode of vibration of the string suggests that inharmonics and sub-harmonics **are** produced.

Such being the case, it was necessary to analyse the records for inharmonics and subharmonics. Such methods of analysis, if great accuracy is desired, are extremely complicated and laborious. The records taken were not accurate to the degree required for such detailed analysis.

So, an easier method due to **Hermann** (modified by **Scripture**) was employed for the present purpose[1]. It is as follows :

Admitting the presence of inharmonics in the tone under consideration, it is obvious that the results of Fourier analysis are really the combined results of both harmonics and inharmonics. If the number and intensities of inharmonics were zero or negligible, then in a harmonic analysis each harmonic will have a definite place of its own, with a definite frequency and amplitude. However, if inharmonics were present, they would have no place of their own in a harmonic analysis but would show themselves in all the harmonics as an intensification of the amplitudes of the harmonics.

Out of this mixed results, therefore, inharmonics have to be separated, for which **Hermann** gives the following rules: (1) When a harmonic of large amplitude appears with neighboring harmonics of very small amplitude, it may be considered alone as indicating the partial. (2) When the strong harmonic is accompanied by two neighboring strong harmonics, all three should be considered. (3) When one of the neighboring harmonics is more than twice as great as the other, only the former should be considered with the strong harmonic.

Scripture points out that such procedure involves too arbitrary a selection of harmonics for inharmonic analysis and proposes the following rule, which is adopted here. "In each case, it is assumed that there were present as many (inharmonic) component tones as there are prominent harmonics in the harmonic plot; then every amplitude that is smaller than both its neighbors is divided into two parts proportional to their sizes; the weighted mean is then calculated for each group of amplitudes round a maximum, the calculation being extended to the parts of the minimum amplitudes." The above gives the ordinal value of the inharmonic. For approximate value of the amplitude of the inharmonic the following rule is suggested. 'When the inharmonic does not differ much from a harmonic, the largest amplitude in the group is taken for that of the inharmonic. When it differs much (for example, when it lies about half-way between two harmonics), 4/3 of the amplitude of the largest harmonic in the group is taken.'

Based on these, the following calculations for each string are given :

1. Sa string with juari :

No. of harmonic :	1	2	3	4	5	6
Amplitude of ,, : (in cms.)	01034.	01249.	01104.	03498.	01979.	049.

There are minima at lst, 3rd and 5th harmonics. The method of calculation will make it clear that, whenever there is a minimum at the first harmonic (fundamental), it is to be neglected as the ordinal number of the inharmonic, if so obtained, will be that of the fundamental itself. So, neglecting the first minimum, only the second and third minima are to be taken into consideration for division of amplitude.

(a) The amplitude of the 3rd harmonic is divided in the ratio of .01249 : 03498, i. e., into .002897 and .008143. Similarly, .01979 is divided in the ratio of .03498: .049, i. e, into .008243 and .01155. Now, around the first maximum we have:

$$\frac{(1\times .01034)+(2\times .01249)+(3\times .002897)}{.01034+.01249+.002897}=1.598;$$ which is the ordinal number of the inharmonic. Hence, its frequency = 115 cps (= 72 × 1.598), and amplitude = $\frac{4}{3}$ × .01249 = .01665 cms.

(b) Around the second maximum:

$$\frac{(3\times .008143)+(4\times .03498)+(5\times .008243)}{.008134+.03498+.00824}=4.002$$

Therefore, frequency = 288.1 cps and amplitude = .03498 cm.

(c) Around the third maximum:

$$\frac{(5 \times .01155) + (6 \times .049)}{.01155 + .049} = 5.809.$$

Therefore, the frequency of the inharmonic = 418.2 cps. and the amplitude .049 × $\frac{4}{3}$ = .065 cms.

2. Ma string with juari :

No.	1	2	3	4	5	6
Amplitude :	.03311	.004336	.02824	.02851	.06215	.04467

.004336 is divided in the ratio of .03311 : .02824, into .00234 and .001996. .04467 is not divided as there is no tone beyond it.

(a) Round the first maximum :

$$\frac{(1 \times .03311) + (2 \times .00234)}{.03311 + .00234} = 1.066.$$

Frequency = 102.3 cps. Amplitude = .03311 cms.

(b) Round second maximum :

$$\frac{(2 \times .001996) + (3 \times .02824) + (4 \times .02851) + (5 \times .06215) + (6 \times .04467)}{.001996 + .02824 + .02851 + .06215 + .04467} = 4.742.$$

Frequency = 455.3 cps. Amplitude = ×.06215 $\frac{4}{3}$ = .08287 cm.

3. Pa string with juari :

No.	1	2	3	4	5	6
Amplitude :	.01287	.01004	.007196	.01226	.01536	.01183

.007196 is divided in the ratio of .01004 : .01226 into .00324 and .003956.

(a) $$\frac{(1 \times .01287) + (2 \times .01004) + (3 \times .00324)}{.01287 + .01004 + .00324} = 1.632.$$

Frequency = 176.2 cps. Amplitude = $\frac{4}{3}$ × .01287 = .01916 cm.

(b) $$\frac{(3 \times .003956) + (4 \times .01226) + (5 \times .01536) + (6 \times .01183)}{.003956 + .01226 + .01536 + .01183} = 4.808.$$

Frequency = 519.2 cps. Amplitude = $\frac{4}{3}$ × .01536. = .02048 cms.

4. Sa_1 string with juari :

No.	1	2	3	4	5	6
Amplitude :	.01032	.02598	.03301	.01364	.01821	.01883

.01364 is divided in the ratio of .03301 : .01321 into .008468 and .005172

(a) $$\frac{(1 \times .01032) + (2 \times .02598) + (3 \times .03301) + (4 \times 008468)}{.01032 + .02598 + .03301 + .008468} = 3.41.$$

Frequency = 489.9 cps. Amplitude = $\frac{4}{3}$ × .03301 = 04401. cms.

(b) $$\frac{(4 \times .005172) + (5 \times .01821) + (6 \times .01833)}{.005172 + .01821 + .01833} = 5.315.$$

Frequency = 765.6 cps. Amplitude = .01833 × $\frac{4}{3}$ = .02444 cms.

5. Sa_2 string with juari:

No. :	1	2	3	4	5	6
Amplitude :	.01477	.01545	.03181	.01587	.01493	.009167

Therefore,

$$\frac{(1\times.01477)+(2\times.01545)+(3\times.03181)+(4\times.01587)+(5\times.01493)+(6\times.009167)}{.01477+.01545+.03181+.01581+.01493+.009167}$$

= 3.276

Frequency = 471.9 cps. Amplitude = $\frac{4}{3}\times .03181 = .04241$ cms.

6. Sa string without juari :

No. :	1	2	3	4	5	6
Amplitude:	.01281	.01792	.004718	.09506	.03972	.03842

.004718 is divided in the ratio of .01792 : .09506 into .000748 and .00397.

(a). $\frac{(1\times.01281)+(2\times.01792)+(3\times.000748)}{.01281+.01792+.000748} = 1.62.$

Frequency = 116.6 cps. Amplitude = $\frac{4}{3}\times .01792 = .02389$ cms.

(b). $\frac{(3\times.00397)+(4\times.09506)+(5\times.03972)+(6\times.03842)}{.00397+.09506+.03972+.03842} = 4.634$

Frequency = 333.7 cps. Amplitude = .1267 cms.

7. Ma string without juari :

No.	1	2	3	4	5	6
Amplitude :	02627	.02385	.003601	.01775	.04274	.0265

.003601 is divided in the ratio of .02385 : .01775 into .002065 and .001536.

(a) $\frac{(1\times.02627)+(2\times.02385)+(3\times.002065)}{.02627+.02385+.002065} = 1.537$

Frequency = 147.5 cps. Amplitude = $\frac{4}{3}\times.02627 = .03503.$

(b) $\frac{(3\times.001536)+(4\times.01775)+(5\times.04274)+(6\times.0265)}{.001536+.01775+.04274+.0265} = 5.065.$

Frequency = 486.3 cps. Amplitude = .04274

8. Pa string without juari

No. :	1	2	3	4	5	6
Amplitude :	.006789	.007271	.02522	.08447	.01429	.003667

$$\frac{(1\times.006789)+(2\times.007271)+(3\times.02522)+(4\times.08447)+(5\times.01429)+(6\times.003667)}{.006789+.007271+.02522+.08447+.01429+.003667} = 3.728$$

Frequency = 402.6 cps. Amplitude = $\frac{4}{3}\times.08447 = .11263$ cms.

9. Sa_2 string without juari :

No.	: 1	2	3	4	5	6
Amplitude :	.01111	.01411	.0257	.01139	.06191	.0108

.01139 is divided in the ratio of .0257 : .06191 into .003341 and .008049.

(a) $\frac{(1\times.01111)+(2\times.01411)+(3\times.0257)+(4\times.003341)}{.01111+.01411+.0257+.003341} = 2.577$

Frequency = 371.1 cps. Amplitude = $\frac{4}{3}\times.0257 = .03428.$

$$\text{(b)}\quad \frac{(4\times.008049) + (5\times.06191) + (6\times.0108)}{.008049 + .06191 + .0108} = 5.032$$

Frequency = 724.7 cps Amplitude = .06191 cms.

The results of the above calculations are summarised in the following table (Table V): See also Fig.XV. and XVI. *

Table V.
Inharmonics.

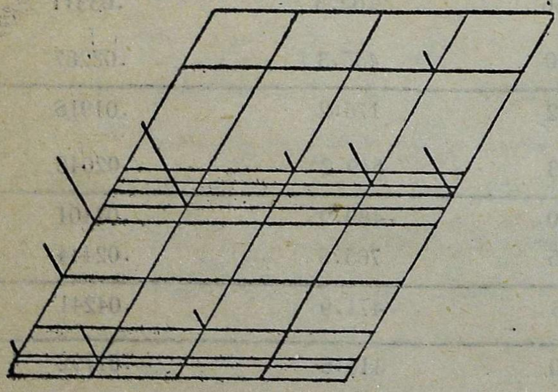
	String	Ordinal Number	Frequency cps	Amplitude cms
With juari	Sa	1.598	115	.01665
		4.002	288.1	.03498
		5.809	418.2	.06533
	Ma	1.066	102.3	.03311
		4.500	455.3	.08287
	Pa	1.632	176.2	.01916
		4.808	519.2	.02048
	Sa_1	3.410	489.9	.04401
		5.315	765.6	.02444
	Sa_2	3.276	471.9	.04241
Without juari	Sa	1.620	116.6	.01792
		4.634	333.7	.1267
	Ma	1.537	147.5	.03503
		5.065	486.3	.04274
	Pa	3.728	402.6	.1126
	Sa_2	2.577	371.1	.03423
		5.032	724.7	.06191

* Strictly, these results have to be included in the calculations for tone-centroid and intensity centroid. However, they are too approximate for such a purpose and, therefore, have not been so included.

It will be seen that in many cases there are two regions of inharmonics of approximate ordinal values of 1.5 and 4.5 ($= 3 \times 1.5$). There is not much difference between with and without juari; without juari the ordinal values are slightly higher than with juari. So that it can be said with a fair degree of approximation that apart from harmonics the following inharmonics (note that they are musically very harmonious) are produced :

String	Note	
Sa	(pa; Ri)	**(chatusruti).**
Sa	(Pa; Ri)	
Ma	(Sa; Pa)	
Pa	(Ri; dha)	**(chatusruti)**

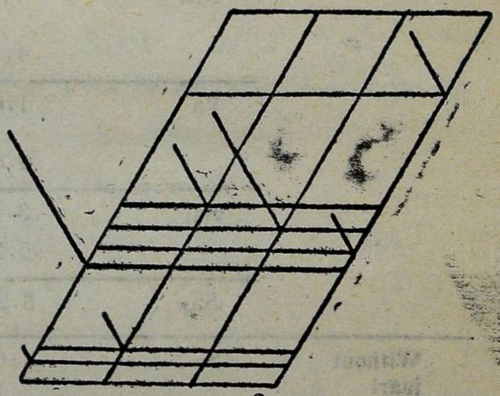
With Juari



Sa Ma Pa Sa_1 Sa_2

Fig. XV. Inharmonic plot, showing the amplitudes of the inharmonics for each string. For details of data see text.

Without Juari



Sa Ma Pa Sa_2

Fig. XVI. Inharmonic plot of amplitudes. Without **Juari**.

Reference :

1. E. W. SCRIPTURE. **Researches in Experimental Phonetics** 77–83 (Carnegie Inst., 1906).

Chapter IV

Subjective Tonal-Spectrum*

Though we have the term **swayambhu** in our musical texts to indicate physical, objective harmonics we do not have any term for subjective tones. So, it is not possible to know if our musicologists ever observed such tones.

The first report of such observation seems to be that of **Sorge,** a German musicologist[1]. Later **Romieu** and specially **Tartini** also reported similar observations. Hence, often such tones are also known as tones of Tartini. Later **Helmholtz** reported the existence of summation tones, in addition to the difference tones and aural harmonics of earlier writers.[2]

At present it is accepted by most investigators that there are mainly two broad categories of tones produced within the ear. (1) aural harmonics and (2) combination tones which are of two kinds - difference and summation tones. **

(1) When a tone of sufficient intensity impinges on the ear, the ear itself produces tones, which can be shown to be absent in the physical tone impressed and which are integral multiples, in frequency, of the impressed tone. Hence they are called aural harmonics.

(2) When two tones (**h** and l) impinge on the ear, tones which are combinations – difference and summations–of the primaries are generated within the ear. These are the combination tones of the value ($mh \pm nl$), **m** and **n** being integers. The order of a combination tone is given as ($m+n-1$).

The cause for the generation of subjective tones has been attributed to the non-linearity of the auditory mechanism, though this has been strongly objected to by some (like Max F. Meyer). That is to say, the response of the auditory mechanism is not proportional to the impressed force (**Hooke's** law becoming invalid), if the impinging tone is of sufficient intensity. The locus of such a distortion is mainly the cochlea, though the middle ear does play a certain part in the process.

Under such a condition of distortion, the relations between the response (X) and the impressed force (p) is given by the equation :[3]

$$X = a_0 + a_1 p + a_2 p^2 + a^3 p^3 \ldots\ldots\ldots a_n p^w.$$

* The term ' subjective tone ' is a traditional one, including in it both aural harmonics and combination tones. The term is, however, objected to as being ' mentallistic ' (whatever that may mean) by psychologists of the ' behaviorist ' school (like for example, Max F. **Meyer**). The only other term suitable to cover both aural harmonics and combination tones may be ' aural tones. ' As yet there is no inclusive term except ' subjective tones, ' which is sufficiently clear, for such tones are subjective, in the sense that they are produced **within** the subject and not **without**. Hence this term is used here, without introducing a new term like ' aural tones. '

** Recently **W. Meyer - Eppler** of Bonn (Germany) has reported the existence of quotient tones which are the H. C. F. of the two primaries, vide., Residual Tone and Formant Tone, **Gravesaner Blatter,** Vol 4. No. 14. 84–109 (1959)

If p represents the sum of two sinusoidal forces, then an expansion of the above equation in terms of their frequencies gives the first four terms as:

$$a_0 = a_0$$

$$a_1 p = a_1 (p_1 \cos w_1 t + p_2 \cos w_2 t)$$

$$a_2(p)^2 = a_2 \Big[\frac{1}{2} p_1^2 \cos 2 w_1 t + \frac{1}{2} p_2^2 \cos 2 w_2 t + p_1 p_2 \Big\{ \cos(w_1 - w_2) t + \cos(w_1 + w_2) t \Big\} + \frac{1}{2}(p_1^2 + p_2^2) \Big]$$

$$a_3(p)^3 = a_3 \Big[(\frac{3}{4} p_1^3 + \frac{3}{2} p_1 p_2^2) \cos w_1 t + \frac{1}{4} p_1^3 \cos 3 w_1 t + (\frac{3}{4} p_2^3 + \frac{3}{2} p_1^2 p_2) \cos w_2 t + \frac{1}{4} p_2^3 \cos w_2 t + \frac{3}{4} p_1^2 p_2 \cos(w_2 t + 2 w_1 t) + \frac{3}{4} p_1^2 p_2 \cos(w_2 t - 2 w_1 t) + \frac{3}{4} p_1 p_2^2 \cos(w_1 t + 2 w_2 t) + \frac{3}{4} p_1 p_2^2 \cos(w_1 t - 2 w_2 t) \Big].$$

There are three methods open for the determination of internal-ear spectrum.

(1) It can be calculated from equations of the above type. Obviously the number of terms in the expression of the equation is a matter of choice, the expansions becoming more cumbersome with increasing number of terms.

The difficulty that is the most limiting is the value of the constants a_0, a_1, a_2, a_3,……a_n involved in equation.

Prof. **E. G. Wever** says, (in a private communication to me, May 3, 1954) regarding these constants :-

" Unfortunately the values of the constants a_0, a_1, a_2, etc., are not known. They are not even universal constants, but their values vary according to the particular conditions of the distortion. Increasing the over-loading of the ear, increases the values of the constants beyond a_0 and a_1 and thus increases the total harmonic content.

" Therefore, it is not possible to predict from the transformation formula what the amounts of the individual harmonics are going to be. The value of the formula is only to predict what the harmonics are. It will be possible, of course, to carry out an experiment in which the magnitudes of the harmonics are determined, and then from these data to calculate what the values of the constants are for this particular case. We have obtained some data of this kind. But it must be remembered that as the conditions of stimulation change, these values of the constants change also.

" From my experience in the study of distortion in the ear I feel that it is unwise

to take the mathematical expression of transformation too seriously. I consider it only an approximate representation of the way in which the ear is operating."

Such being the case, any calculated value in the subjective tonal spectrum is bound to be very approximate.

(2) The experimental method usually used with human subject is called the method of best beats, or the exploring - (probe -) tone method.[4]

This method consists in introducing a third (exploring - or probe-) tone, the frequency and intensity of which are adjustable. Now, if the frequency of the exploring tone is near that of a subjective tone, prominent beats will be heard. Further, the beats are heard best when the intensities of the beating tones are equal. Hence, the intensity of the exploring tone is adjusted till the beats are heard best and the intensity of the explorig tone can be taken to be equal to that of the subjective tone.

Wegel and **Lane** have used this method successfully to study the interaction of two tones.[5] **Cotton**[3] used it to find subjective tones between unision and octave. **Fletcher** has constructed graphs to show the inter-relations of the magnitudes of the fundamentals and the aural harmonics, based on the data obtained from such experiments.[4]

(3) The most common method employed now a days is the analysis of cochlear potentials, specially in animals.

The study of cochlear microphonics was brought into great prominence by the work of **Wever** and **Bray**.[5] They reported that 'after placing electrodes upon the medulla or the auditory nerve of a decerebrate cat, they were able, by listening with telephone receivers to the amplified signals, to recognize, not only pure tone used as stimuli, but even words spoken to the cat !'[6]

These microphonics are distortion-potentials and are the transformation of the mechanical energy of the sound into electrical energy without the contribution of energy by the tissue itself; they are not the action potentials of the nerve-cells.

This phenomenon has been taken advantage of in the study of auditory-nerve response. When two pure tones stimulate the ear simultaneously, the cochlear microphonics, on analysis, reveal the harmonics and combination tones of the two primaries. **Newman, Stevens** and **Davis** report the existence of 66 tones of this nature produced from two primaries ! [7]

(At the time of the experiment reported herein, the necessary equipment and facilities for the exploring-tone determination of tonal spectrum were not available to me. Hence, I give below an **approximate** estimate of the subjective-tone spectrum from the formula given above. Such equipment is, however, now available to me and I expect to report the result of the investigation soon).

The limitations of the calculated tonal spectrum (subjective) are indeed great. For (1) the characteristic (response) curve of the ear varies with each individual (2) the constants a_0, a_1, a_2, ... also have no fixed value and vary with the experimental conditions.

However, an overall characteristic curve would give a fairly approximate estimate of a 'normal' ear. The constants a_0, a_1, a_2 ...... calculable from such a curve would also be approximate values for 'normal' ears under 'normal' conditions. (The 'normal' states of one individual and experimental conditions are obviously hypothetical and indicate only an abstracted condition).

Such an 'overall' characteristic curve has been presented by **Stevens** and **Newman** as shown in Fig. XVII. The characteristics a_0, a_1 and a_2 calculated from this have the following values for the right quadrant.*

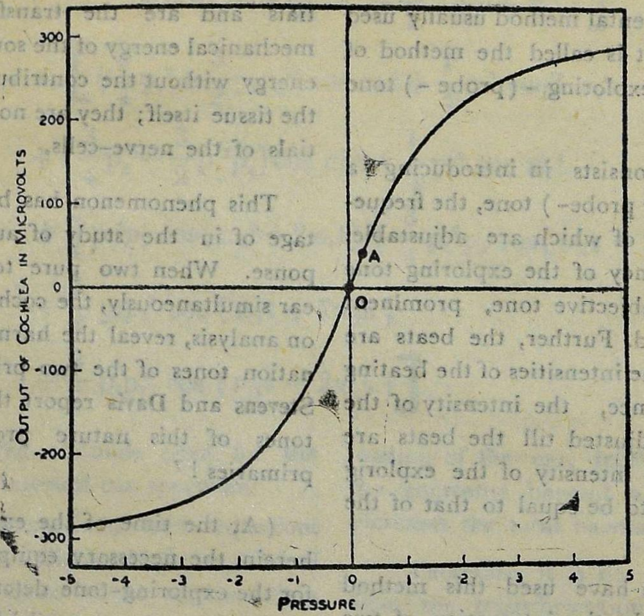


Fig. XVII. 'Approximate characteristic curve of the ear. The curve relates the electric potential of the cochlear microphonics (ordinate) to the applied sound-pressure in dynes per square centimeter. Thus when the pressure varies sinusoidally about the zero value, the instantaneous voltage in the cochlea may be read from the curve and the resulting wave-form determined. Factors such as tension on the muscles of the middle ear may shift the operating point (point corresponding to zero pressure and zero voltage) from **O** to some other point such as **A**. With the operating point at **A**, both odd and even harmonics are present in the output.' (Reproduced by permission, from S. S. STEVENS and E. B. NEWMAN, On the Nature of Aural Harmonics, **Proc. Nat. Acad. Sci.**, 22. 1936. Photographed from S. S. STEVENS and H. DAVIS, **Hearing** 195, John Wiley, 1947).

* My thanks are due to Dr. **A. D. Taskar**, my former colleague at the Deccan college and now Statistician, Nutrition Research Institute, Coonoor, for calculating these constants.

(a_0 = average of y values. (Not required for calculations)

$a_1 = 0.1357$

$a_2 = -0.0429$

It must be noted that the curve is a pressure-electric potential relation. If the curves were expressed in terms of amplitudes of stimulus and response it would be of a different magnitude and nature; but the values of the constants would bear the same interrelation. In the following calculations amplitudes have been used instead of pressure values. This change would mean no difference, as the interrelation of tones obtained would remain the same, except that they have not been multiplied by the constant of proportionality between amplitude and pressure.

Now, in the estimation of the subjective tones (up to the third term) each pair of tones in the objective tonal spectrum has to be taken in turn. But, it would be redundant if each tone from each string be considered; for, it is obvious that many of these tones, are of the same frequencies (though produced from different strings). For instance the **Sa** produced from **Sa** and Sa produced from **Sa**$_1$ and Sa$_2$ strings have the same frequency. All these (and probably Sa produced from **Pa**) would add up in their energy and we may say, to a first approximation without taking into account any phase differences, that the total intensity of **Sa** (objective) will be proportional to the sums of the squares of amplitudes of the **Sa** produced from each string. These values (Σc^2) are given in Table IV.

Now, to obtain the amplitude of any particular tone in the objective (external-ear) spectrum, we may take the square root of this cumulative value (for that frequency). This I shall term as the **effective amplitude**, A. Thus, at any frequency :

$$A = \sqrt{\Sigma c^2}$$

Where $\Sigma c^2 = c_1^2 + c_2^2 + c_3^2 \ldots\ldots (c_1, c_2, c_3$ being the amplitudes of tones of the **same** frequency produced from different strings).

These values of As may now be substituted, in the equations for **p** and the resultant amplitudes will be those of subjective tones, which may be termed as **s**.

The effective, amplitude of each tone in the objective spectra is given in the following table :-

Table VI

Effective Amplitude

Note	Frequency cps	A in cms
Pa-tuning :-		
Sa	72	.01033
Pa	108	.01287
Sa	144	·02193
Pa	216	·01519
Sa	288	·04623
Ri	324	·007196
Ga	360	·01982
Pa	432	·06822
Ni	540	·01535
Sa	576	·02092
Ri	648	·01183
Ga	720	·02355
Pa	864	·02351
Ma-tuning :-		
Sa	72	·01033
Ma	96	·03311
Sa	144	·02193

(contd.).

Table VI contd.

Ma	192	·004252
Pa	216	·01104
Ṡa	288	·05418
Ġa	360	·01982
Ṁa	384	·02851
Ṗa	432	·06710
Ḋha	480	·05880
S̈a	576	·04933
G̈a	720	·02355
P̈a	864	·02351

The subjective amplitude of a tone being s, the subjective energy of the tone wil be proportional $s^2 = S$. The cumulative $S = \Sigma S = \Sigma s^2$ for each frequency from various subjective tones can then be estimated and a graph constructed for internal ear spectrum.

The various orders of tones obtainable from the first three terms of the transformation equation are listed below :

l = lower tone h = higher tone.

	Aural harmonics	**(Combination tones)**
First term:	nil	nil
Second term:	l, h.	nil
Third term:	2l, 2h.	$l \sim h$, l + h.

The third term generates the first order combination tones and second (aural) harmonics Further terms become extremely complicated. (**vide, Cotton's** paper referred to in ft. nt. 3). Even the calculation of a_3 and subjective tones of tones of second order are very laborious, and, hence, the calculations have been confined to those of first order. The detailed calculations are given in Appendix II, as a sample, wherein the amplitudes and frequencies are tabulated as follows:

Table VII
Subjective Tones

No.	Primaries (Frequency and A)	Aural Harmonics No.	Frequency	Note	s	S
...	...(...)	1	...	...	...	...
	......	2	...	...	...	...
...	...(...)	1	...	...	...	...
	......	2	...	...	...	...
...	...(.)	1	...	...	...	...
	......	2	...	...	...	...

Combination of .... (...). A = ... with:

No.	Primaries (Frequency and A)	Combination Tones Order	Frequency	Note	Ratio	s	S
...	...(...)	h − l	....	....	....	....	....
	......	h + l	....	....	....	....	....
...	...(...)	h − l	....	....	....	....	....
	......	h + l	....	....	....	....	....
...	...(...)	h − l	....	....	....	....	....
	...(...)	h + l	....	....	....	....	....

For actual values see Appendix II

By the transformation formula herein employed, the magnitudes of the summation tones are the same as the difference tones, both being expressed as $a_2 . p_1 . p_2$. **Fletcher** has computed the internal ear spectrum of an organ tone on similar assumptions. He further assumes that this magnitude is of the same order as that of the corresponding aural harmonic.[9] This is approximately true, but in the present work, the combination tones and aural harmonics have be treated separately.

The results of actual experiments, though not consistent, are the same as these inferences. **Wever** and **Laurence** found that the response X is proportional to the intensities of both the primaries, **in difference as well as summation tones,** of the first order. That is $X(h-l) = K_1 . P_h . P_l$. In general, $X(mh \pm nl) = KP_h^m P_l^n$, where P_h and P_l are the forces due to the two primaries h and l, m and n are integers. The experimental method used by them was that of analysis of cochlear microphonics. They do not give any such relationship for the aural harmonics.[10]

On the other hand, **Moe,** using the exploring tone method, reports that in general **summation tones were less intense** than the corresponding difference tones of the same order.[11]

It was already pointed out that the transformation formula was only an approximate representation of the actual auditory mechanism. Further, as Prof. **Wever** points in his letter quoted, the values of the constants themselves change according to the experimental conditions, specially the intensity of stimulus. Thirdly, the calculated values presented here are for a hypothetical, ' abstracted ' ear. Actuality is, therefore, bound to be different from such an abstract picture. Lastly, in the calculations made, no account has been taken of masking, phase-effects and higher order tones.[12] It will be noted that many of the subjective tones are near enough in frequency and intensity to cause prominent beats. This, therefore, complicates the tonal structure further. Hence, the internal ear spectrum presented here is only an **approximation** to actuality. It will be later on pointed out that besides the physical and neurophysiological problems so far discussed, there are the questions of conscious attention and choice and unconscious repression and artificial tone colours.

Albeit an approximate picture, the internal ear spectra may be compared with the external-ear spectra. In Table VIII are brought together the details, for **Pa** - and **Ma** - tuning. The musical signature (Note) and the interval for each note in its own octave are given in the first two columns. The fourth column (ΣSn^2) gives the ' subjective quantity ' proportional to ' subjective intensity '. (It is doubtful whether the latter would correspond to loudness; loudness is a quantity with many different characteristics than the ' subjective intensity ' herein described). The last column gives these values in ratio to the ΣSn^2 of **Sa**, to facilitate comparison. The last are plotted in the internal ear spectra. (Figs. XVIII & XIX)

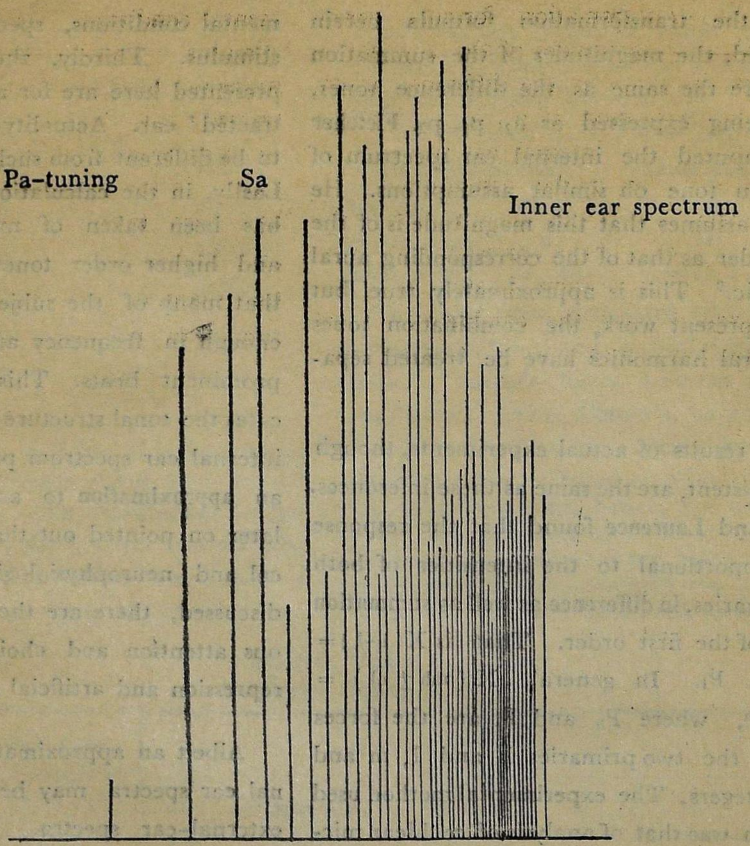


Fig. XVIII. In this and the following inner ear spectra, the abscissa is the logarithm of frequency or musical interval and the ordinate the logarithm of the energy ratios. In all cases a constant has been aded to all the log. energy ratios to make all of them positive. More conventionally, these logarithmic relations could be expressed in terms of dbs, taking the 'energy' of Sa to be zero decibel.

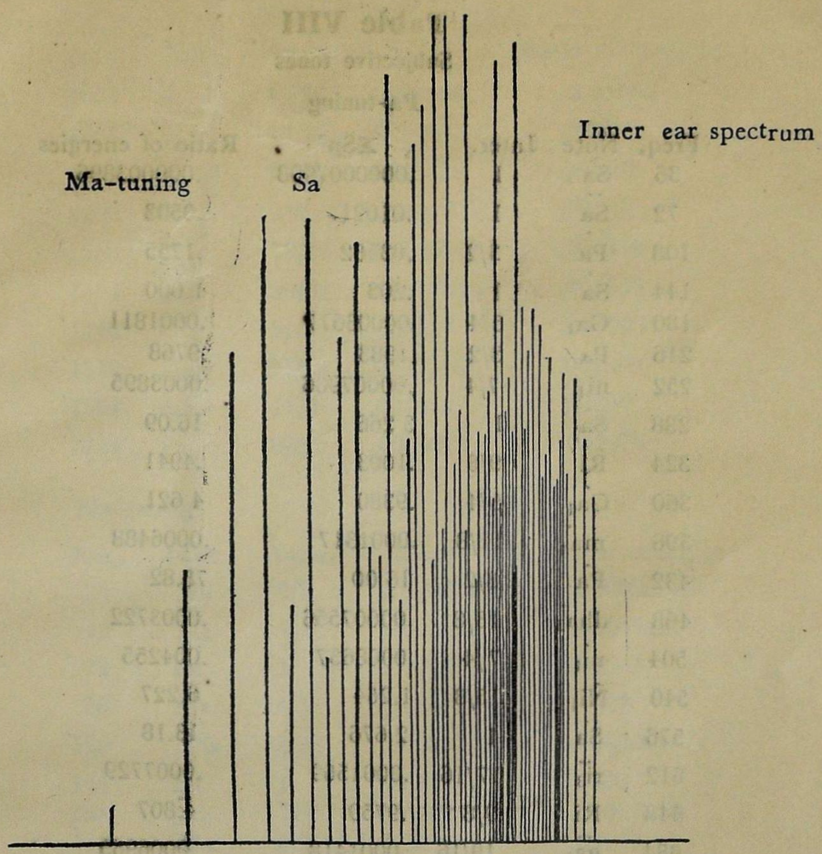


Fig. XIX

Unless otherwise specified all the internaal ear spectra to be presented hereonwards, are in terms of the logarithms of the quantities involved; for it is known that the physiological processes of reception and perceptiona are a great extent logarithmic (Weber-Fetchner law). Hence such a representation would give a closer perspective to actuality than linear ratios.

Comparison of objective (external) and subjective (internal-ear) spectra.

It will be seen from the details in Appendix II, that wherever there is a frequency of an objective tone, the subjective magnitude of such a frequency is mainly that of the subjective fundamental. Compared to this the contribution to the magnitude by the second aural harmonic and combination tones is almost negligible.

Table VIII

Subjective tones

Pa-tuning

Freq.	Note	Inter.	ΣSn^2	Ratio of energies
36	Sa	1	.0000007908	.000003896
72	Sa	1	.01021	.0503
108	Pa	3/2	.03562	.1755
144	Sa	1	.203	1.000
180	Ga_1	5/4	.00003677	.0001811
216	Pa	3/2	.1983	.9768
252	ni_1	7/4	.00007906	.0003895
288	Sa	1	3 266	16.09
324	Ri	9/8	.1003	.4941
360	Ga_1	5/4	.9380	4 621
396	ma_1	11/8	.0001317	.0006488
432	Pa	3/2	16.00	78.82
468	dha_3	13/8	.00007556	.0003722
504	ni_1	7/4	.0008637	.004255
540	Ni_1	15/8	1.264	6.227
576	Sa	1	2.676	13.18
612	ri_3	17/16	.0001569	.0007729
648	Ri	9/8	.9759	4.807
684	ga_2	19/16	.0001418	.0006985
720	Ga_1	5/4	5.303	26.12
756	Ga_3	21/16	.0004292	.002114
792	ma_1	11/8	.0002440	.001202
828	ma_3	23/16	.0007514	.003701
864	Pa	3/2	7.607	37.47
900	dha_1	25/16	.0001717	.0008458
936	dha_3	13/8	.001061	.005227
972	Dha_2	27/16	.0002079	.001024
1008	ni_1	7/4	.00762	.03754
1044	ni_2	29/16	.00005782	.0002828
1080	Ni_1	15/8	.002524	.01243
1116	Ni_3	31/16	.0002368	.001167
1152	Sa	1	.01095	.05394

(cont.

Table VIII (contd)

Freq.	Note	Inter.	ΣSn^2	Ratio of energies
1188	ri_1	33/32	.0001596	.0007862
1224	ri_3	17/16	.0007671	.003779
1260	ri_5	35/32	.0003826	.001885
1296	Ri	9/8	.001078	.00531
1368	ga_2	19/16	.0002676	.001318
1404	ga_4	39/32	.0004734	.002332
1440	Ga_1	5/4	.001217	.005995
1512	Ga_3	21/16	.0003246	.001599
1584	ma_1	11/8	.001415	.00697
1728	Pa	3/2	.0004213	.002075

Ma-tuning:

Freq.	Note	Inter.	ΣSn^2	Ratio of energies
24	Ma	4/3	.0000004648	.000002523
48	Ma	4/3	.0000838	.0004549
72	Sa	1	.01021	.05543
96	Ma	4/3	.1884	1.023
120	Dha	5/3	.00003797	.0002061
144	Sa	1	.1842	1.000
168	ga_1	7/6	.00001163	.00006314
192	Ma	4/3	.01335	.07247
216	Pa	3/2	.1049	.5695
240	Dha_1	5/3	.0001298	.0007047
264	ni_3	11/6	.0001096	.000595
288	Sa	1	4.486	24.35
312	ri_4	13/12	.00003942	.000214
336	ga_1	7/6	.001467	.007964
360	Ga_1	5/4	.938	5.092
384	Ma	4/3:	2.207	11.98
408	ma_2	17/12	.0001019	.0005532
432	Pa	3/2	15.47	83.98
456	dha_2	19/12	.000198	.001075
480	Dha_1	5/3	14.68	79.7
504	ni_1	7/4.	.0008324	.004519

(contd.)

Table VIII (contd.)

Freq.	Note	Inter.	ΣSn^2	Ratio of energier
528	ni_3	11/6	.002732	.01483
552	Ni_2	23/12	.0001901	.001032
576	Sa	1	14.78	80.24
600	ri_2	25/24	.00006552	.0003557
624	ri_4	13/12	.001686	.009153
648	Ri	9/8	.001609	.008735
672	ga_1	7/6	.004256	.02311
696	ga_3	29/24	.0003759	.002041
720	Ga_1	5/4	5.308	28.82
744	Ga_2	31/24	.0003255	.001767
768	Ma	4/3	.02009	.1091
792	ma_1	11/8	.002522	.01369
816	ma_2	17/12	.00523	.02839
840	ma_4	35/24	.001685	.009148
864	Pa	3/2	7.622	41.38
912	dha_2	19/12	.02384	.1294
936	dha_3	13/8	.001746	.009479
960	Dha_1	5/3	.009456	.05134
1008	ni_1	7/4	.02375	.1289
1056	ni_3	11/6	.01731	.09397
1080	Ni_1	15/8	.0006129	.003327
1104	ni_1	23/12	.001012	.005494
1152	Sa	1	.008191	.04447
1200	ri_1	25/24	.0005087	.002762
1224	ri_3	17/16	.0005998	.003256
1248	ri_4	13/12	.001289	.006998
1296	Ri	9/8	.005786	.03141
1344	ga_1	7/6	.0006358	.003452
1440	Ga_1	5/4	.005435	.02951
1584	ma_1	11/8	.001415	.007682
1728	Pa	3/2	.0004213	.002287

Hence, the inter-relationship of such frequencies in magnitudes is **almost** the same objectively and subjectively (compare Tables IV and VIII). However, subjectively in both Ma- and Pa tunings, the Sa (144) is slightly stronger than objectively; this is evident in that the energy ratio of subjective tones to Sa (= 1) is less than objective tones.

The first thing that strikes one is the difference in the number of tones in both the spectra. In the external-ear spectra, both in Pa and Ma tuning, there are 13 tones. Whereas in the internal-ear spectra Pa-tuning has 42 tones and Ma-tuning has 52 tones !

Again in the external spectrum of (1) Pa-tuning there are only five different intervals (Sa = 1, Ri = 9/8, Ga = 5/4, Pa = 3/2 and Ni = 15/8) and (2) Ma-tuning has also only five (Sa = 1, Ga = 5/4, Ma = 4/3, Pa = 3/2, Dha = 5/3). But the internal ear spectra of Pa-tuning shows 19 different intervals and Ma-tuning shows 21 different intervals. In all, both tunings give together 31 intervals (Table IX).

Table IX

Intervals due to both tunings

No.	Note	Interval
1	Sa	1
2	ri_1	33/32
3	ri_2	25/24
4	ri_3	17/16
5	ri_4	13/12
6	ri_5	35/32
7	Ri	9/8
8	ga_1	7/6
9	ga_2	19/16
10	ga_3	29/24
11	ga_4	39/32
12	Ga_1	5/4
13	Ga_2	31/24
14	Ga_3	21/16
15	Ma	4/3
16	ma_1	11/8
17	ma_2	17/12
18	ma_3	23/16
19	ma_4	35/24
20	Pa	3/2
21	dha_1	25/16
22	dha_2	19/12
23	dha_3	13/8
24	Dha_1	5/3
25	Dha_2	27/16
26	ni_1	7/4
27	ni_2	29/16
28	ni_3	11/6
29	Ni_1	15/8
30	Ni_2	23/12
31	Ni_3	31/16

Within the internal ear spectra themselves there are many tones (those that have no external correspondent) which are very weak compared to Sa (and others that have external correspondent). Some of them are a few millionths less intense than Sa. Therefore, it is possible that these may not be heard at all due to masking. On the other hand there is every possibility - and this gives them sufficient importance - that they may be brought into prominence by the musician himself, when he produces tones which have the same or very near frequencies as these. Thus they may be perceived by resonance or by beats, drawing attention to themselves.

Further, the internal-spectra show that tone for tone the fundamental Sa (**madhya Sa - ādhāra sadja**) is not the strongest. Question, therefore, arises as to the usefulness of such a drone that does not bring into prominence the key-note.

The answer to this query should be sought not in the magnitude of a tone itself, but in the phenomenon of octave relations. It is a well-known and recognized fact that the octave relation is one of the most 'fundamental' one in music. It is so 'basic' that tones which are related as octaves are treated being the 'same'. They may vary in the 'pitch' (frequency) scale, but have the 'same tone - colour'. This phenomenon has been a very difficult one to explain, the most convincing of such explanations being those of the two-component theory of **Revesz**[13]. However, the discussion of such theories is irrelevant to the present context ; it is sufficient to note the 'identity' of octaves.

Such being the case, the production and perception of any tone will be greatly helped when there is not only an external auxiliary tone (the drone) of the **same pitch**, but also its octave or octaves. In fact, the fundamental may even be absent almost completely, such octaves being present. So, the 'magnitude' of a tone of the same 'colour' is more important than its actual value in the 'pitch' scale, in this context.

Thus the strength of any note in the drone may be taken to be the sum of all S of the same musical interval, e. g., the strength of 'Sa' will be, the sum of the subjective energies of Ṣa, Sa, Ṡa, S̈a, etc., and that of 'Ri' will be the sum of subjective energies of Ṛi, Ri, Ṙi, R̈i, etc., I have termed this as the **intervallic strength.** (Though as physical quantities such values are not strictly 'additive', being the values at different frequencies, psychologically they may be considered to be so in the sense that the greater the intensity of tone the greater will be its help in the perception and production of its unision and octaves).

The intervallic strengths of the various notes are given in Table X.

Table X

Pa-tuning

Note	Inter.	Inter. strength	Ratio of Inter. strengths
Sa	1	6.166	1.000
ri_1	33/32	.0001596	.00002545
ri_3	17/16	.000924	.0001499
ri_5	35/32	.0003826	.00006205
Ri	9/8	1.077	.1747
ga_2	19/16	.0004094	.0000664
ga_4	39/32	.0004734	.00007678
Ga_1	5/4	6.242	1.012
Ga_3	21/16	.0007538	.0001223
ma_1	11/8	.001791	.0002905

(contd.)

Table X (contd.)

Note	Inter.	Inter. strength	Ratio of Inter. strength
ma_3	23/16	.0007514	.0001219
Pa	3/2	23.84	3.866
dha_1	25/16	.0001717	.00002785
dha_3	13/8	.001137	.0001844
Dha_2	27/16	.0002079	.00003372
ni_1	7/4	.008563	.001389
ni_2	29/16	.00005782	.000009377
Ni_1	15/8	1.267	.2055
Ni_3	31/16	.0002368	.0000384

Ma-tuning:

Note	Inter.	Inter. strength	Ratio of Inter. strengths
Sa	1	19.47	1.000
ri_2	25/24	.0005742	,00002949
ri_3	17/16	.0005998	.00003081
ri_4	13/12	.003023	.0001553
Ri	9/8	.007395	.0003798
ga_1	7/6	.00637	.0003272
ga_3	29/24	.0003759	.00001931
Ga_1	5/4	6.251	.3211
Ga_2	31/24	.0003255	.00001672
Ma	4/3	2.429	.1248
ma_1	11/8	.003937	.0002022
ma_2	17/12	.005332	.0002739
ma_4	35/24	.001685	.00008654
Pa	3/2	23.2	1.192
dha_2	19/12	.02404	.001235
dha_3	13/8	.001746	.00008968
Dha_1	5/3	14.69	.7545
ni_1	7/4	.02458	.001262
ni_3	11/6	.02015	.001035
Ni_1	15/8	.0001629	.000008367
Ni_2	23/12	.001202	.00006174

The ratio of each intervallic strength to that of Sa (ādhāra sadja) – the intervallic ratio to Sa – is shown in the last columns of the Table and in Figs. XX & XXI.

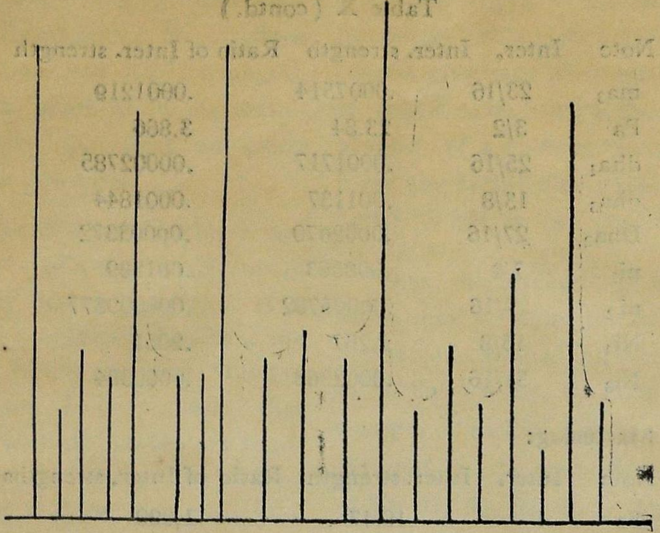


Fig. XX. Inner ear spectrum of intervallic strengths. Pa-tuning.

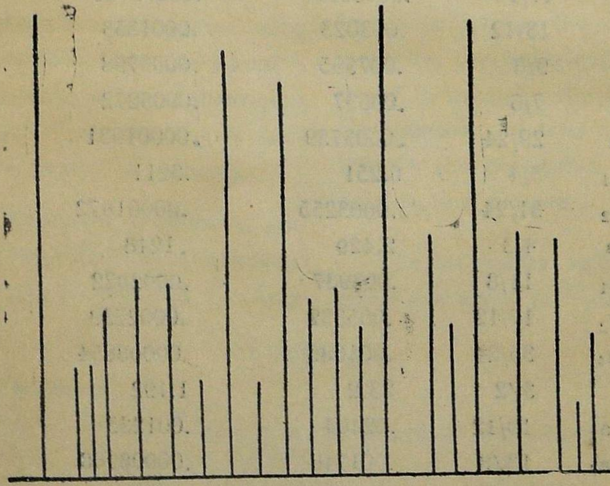


Fig. XXI. Inner ear spectrum of intervallic strengths. Ma-tuning.

It is now seen that 'Sa' emerges as one of the strongest notes*. In Pa-tuning only Pa is stronger than Sa, and that too about four times, Ga being almost equal to Sa. In

* It is necessary to differentiate between a tone and a note. The former is a 'psycho-physical reality'; it is a sound. The latter is a signature or symbol indicating a musical relation of tones.

This is analogous to the distinction we make between 'Phonetics' and 'Phonemics' cf. our discussion of 'reversibility' of speech at these two levels in the paper (with Prof. C. R. SANKARAN and P. C. GANESH-SUNDARAM), Reversed Speech and the Alpha phoneme Theory, **BDCRI** 17. 1-14.(1955) (155)

Ma-tuning Sa is definitely one of the strongest (it occuring as one of the harmonics of Ma), next to Pa by a very small margin. But, Pa is still stronger than Ma. This may be due to the quality of the instrument studied here; it may be also due to the fact that in the combination tones (at least to the first order), Pa occurs more than Ma. In any case, the fundamental (ādhāra sadja) finds great emphasis in both tunings; the other tones lend the necessary tone colour to the backgronnd, thus serving as a very good drone.

Notes and References :

1. Max F. MEYER, **How We Hear and How Tones Make Music** 62 (Branford, 1950).
2. MEYER has commented on this (**ibid** 71) to the effect that these were not really cochlear in origin but could be demonstrated to be present in the objective tone presented.
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   i. The Locus of Distortion in the Ear, **JASA** 11.427 ff. (1940)

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5. R. L. WEGEL and C. E. LANE, The Auditory Masking of one Pure Tone by another and its Probable Relation to the Dynamics of the Inner Ear, **Physical Rev.** 23. 266-85.
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7. E. B. NEWMAN, **et al**, Factors in the Production of Aural Harmonics and combination Tones, **JASA** 9.107-18 (1938).
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11. C. R. MOE, An Experimental Study of Subjective Tones Produced within the Human Ear, **JASA** 14.163 (1943).
12. J. D. TRIMMER and F. A. FIRESTONE, An Investigation of Subjective Tones by means of the Steady Tone Phase Effect, **JASA** 9.23 ff (1938).

    E. K. CHAPIN and F. A. FIRESTONE The Influence of Phase on Tone Quality and Loudness, **JASA** 15.173--80 (1944).
13. G. REVESZ, **Introduction to the Psychology of Music** Ch. 4 (Longmans, 1953).

Appendix II (Sample data)

Subjective tones : **Aural Harmonics.** Pa-tuning

Primaries (Frequency & Eff. Amp.)	Aural Harmonics				
	No.	Freq.	Note	Amplitude = S.*	s² = S**
Sa (72)	1	72	Sa	1402	1964000
A = .01033	2	144	Sa	-2.289	5.236
Pa (108)	1	108	Pa	1747	3051000
A = .01287	2	216	Pa	-3.554	12.63
Sa (144)	1	144	Sa	4736	9764000
A = .02193	2	288	Sa	-28.81	830.3
Pa (216)	1	216	Pa	2061	4248000
A = .01519	2	432	Pa	-4.949	24.49
Sa (288)	1	288	Sa	6273	39360000
A = .04623	2	576	Sa	-22.98	527.9
Ri (324)	1	324	Ri	976.6	953700
A = .007196	2	648	Ri	-1.111	1.234
Ga (360)	1	360	Ga	2689	7235000
A = .01982	2	720	Ga	-8.428	71.03
Pa (432)	1	432	Pa	9258	85700000
A = .06822	2	864	Pa	-99.97	9994
Ni (540)	1	540	Ni	2083	4334000
A = .01535	2	1080	Ni	-4.015	16.12
Sa (576)	1	576	Sa	2839	8058000
A = .02092	2	1152	Sa	-9.387	88.1
Ri 648)	1	648	Ri	1524	2320000
A = .01128	2	1296	Ri	-2.704	7.311
Ga (720)	1	720	Ga	3196	10210000
A = .02355	2	1440	Ga	-19.9	141.6
Pa (864)	1	864	Pa	3191	10180000
A = .02351	2	1728	Pa	-11.86	140.7

* Multiply all vallues by 10^{-6} ** Multiply all values by 10^{-12}

Appendix II

Subjective tones: **First order combination tones.** Pa–tuning.
Combinations of Sa (Frequency = 72 cps and A = .01033) with:-

Primaries (Frequency & Eff. Ampl.)	Combination tones					
	Comb.	Freq.	Note	Inter	s*	S**
Pa (108)	h−1	36	Sa	1	−5.703	3.253
A = .01287	h+1	180	Ga	5/4		
Sa (144)	h−1	72	Sa	1	−9.716	9.441
A = .02193	h+1	216	Pa	3/2		
Pa (216)	h−1	144	Sa	1	−6.730	4.529
A = .01519	h+1	288	Sa	1		
Sa (288)	h−1	216	Pa	3/2	−20.48	41,94
A = .04623	h+1	360	Ga	5/4		
Ri (324)	h−1	252	ni	7/4	−3.188	1.016
A = .007196	h+1	396	ma	11/8		
Ga (360)	h−1	288	Sa	1	−8.782	7.713
A = .01982	h+1	432	Pa	3/2		
Pa (432)	h−1	360	Ga	5/4	−30.23	91.37
A = .06822	h+1	504	ni	7/4		
Ni (540)	h−1	468	dha	13/8	−6.801	4:626
A = .01535	h+1	612	ri	17/16		
Sa (576)	h−1	504	ni	7/4	−9.268	8.59
A = .02092	h+1	648	Ri	9/8		
Ri (648)	h−1	576	Sa	1	−4.974	2.474
A = .01128	h+1	720	Ga	5/4		
Ga (720)	h−1	648	Ri	9/8	−10.43	10.89
A = .02355	h+1	792	ma	11/8		
Pa (864)	h−1	792	ma	11/8	−10.42	10.85
A = .02352	h+1	936	dha	13/8		

* Multiply all values by 10^{-6} ** Multiply all values by 10^{-11}

Chapter V

Review of the Work.

It was an intense awareness of a vacancy due to lack of living contact with the immediate, with little strictness of methodology and much less intuition in most of the works of our predecessors in the so-called field of musical research (!) that impelled us to study the nature of music at various levels, feeling the necessity for a close study of current music in its various aspects. The present essay is a report of some of our preliminary findings in this direction.

The bias of this work was more towards the results of reception and perception of tonal stimulus - the drone. The **psychology** of the drone itself is a very important aspect in our music, and hence this research was mainly directed towards psycho-physical phenomena. This would involve an analysis of the stimulus, response, perception and etc. As characteristic of our researches we started with perception (this was already discussed in the various papers of the present writer already referred to). This thesis is a detailed summary of the analysis of the stimulus and neurophysiological correlates thereof.

Though the actual physical processes which are the cause of the characteristic tone of the tambura are extremely interesting, they are not immediately relevant to the present work and hence are reported only in outline[1].

Considering the fact that the drone is the acoustical correlate of the tonus, it should form a very good **ādhāra s'adja**. It should also create a very good tonal background for tonal blending, contrast and intonation.

At first it appears that the **ādhāra s'adja** from the **tambura** is not very strong, as it should be, if it is to form a good tonic. In the objective (external-ear) spectrum, it is not the strongest tone; naturally, therefore, it is weak in the internal-ear spectrum as well. This is owing to the fact that in both the **Sa** strings the fundamental is very weak in comparison to the harmonics[2]. The weakness of the fundamental is characteristic not only of the **tambura**, but almost any vibrating string. In voices it is so low in intensity that it can be considered to be absent almost. In reproducing sound, as in gramophones, it is possible to completely eliminate the tonic physically and yet the ear itself supplies it by combination tones. In the case of the **tambura** herein studied, even in the internal-ear spectrum the note **Sa** is still poor in magnitude compared to many other notes.

However, in the last chapter, it was suggested that the octave relations help in tonal perception. By what I have termed as **intervallic strength 'Sa'** is definitely a strong note, specially in **Ma**-tuning. In **Pa**-

tuning it is the next strongest to **Pa**. (In any other tambura the inter-relations of notes may come out slightly different). So, it is seen that the drone gives a very good tonic to the musician.

As a tonal background the instrument is eminently satisfactory. Taking into account only the first six harmonics of each string, in the objective spectrum itself there are 13 tones in each tuning. In the internal-ear spectrum, **Pa** tuning gives 42 and **Ma** tuning gives 52 tones. As intervals **Pa**-tuning gives 19 and **Ma** tuning 21. It must be remembered that only the first six objective harmonics have been taken into account. Similarly, only the first two aural harmonics and the first order combination tones have been evaluated. If we take higher harmonics both, objectively and subjectively, and higher order of combination tones, we would find even a richer tonal spectrum.

It is true, as pointed out in the previous chapter, that many of the tones, specially of higher frequencies, have extemely small magnitudes in comparison to the lower ones. This would mean that in all probability they are not heard due to masking. But when a musician introduces a note of these frequencies or near about these values – either as fundamentals or as harmonics, – they may acquire sufficient prominence by resonance or beating. (As a matter of fact this is a very beautiful experience which can be directly sought for).

This naturally leads to choice and attention to tones. For just as a musician can introduce a tone of his choice, so he can choose to attend any tone either of his own music or of the drone It is even possible to divert attention from the strongest of tones of the drone and not hear it at all, thus psychologically 'masking' it. One can almost wholly erect a tonal background of one's choice.

The sources of such choice may be determined by various factors. The first obvious one is the existence of physical stimulus out of which a choice can be made. Secondly, there are the neuro-physiological processes that modify the stimulus, impressing their own characteristics. Thirdly, there are the conscious operation of choice and attention. Finally there are the unconscious and super-conscious sources and motives which affect and determine such choice.

For it is true that unconscious processes do affect to a great extent tonal perception[3]. As in all aspects of life, in music (tonal perception) also, what we attend to is only a very minor part of the outside. The rest of the outside, though not 'perceived' by the conscious mind, are there, forming a peripheral background to the 'perceived' 'central' aspect, lending to the experience a 'plastic quality'. These peripheral backgrounds are obviously 'perceived' by non-conscious levels; if they are capable of being brought into concious focus, they are only at the preconscious levels. But, they may be of such a character that even by tremendous efforts of consciousness they can not be 'called up'; they are then at deeply unconscious levels,

perhaps beyond all conscious study, for ever remaining an 'unqualifiable' experience affecting our perception we do not know how.

Finally there are the superconscious sources and motives which will determine tonal experience. It is not easy to convince a 'modern' of such thing, though such attempts to convince are not necessary. Such knowledge has now become rare and difficult to acquire. If any proof is needed at all at the levels of 'modern' scientific thinking there are the findings of parapsychological researches*. We did have in our country, and even now there are, people who devoted themselves to such study and it is time we took them more seriously.

In brief, music, like any other human experience, is 'multiordinal'. Though it is a unitary experience, we may, for convenience in study, think of it in various 'levels' somewhat as in the following scheme :

Nāda	Physical (acoustical) ↑↓	physical characteristic (the movement of matter) : frequency, intensity, harmonics (inharmonics), phase, etc.
	Physiological (neural). ↑↓	transformation of physical stimuli by physiological operation – pitch, loudness, tonal-quality, subjective tones, etc.
	Psychological ↑↓	conscious and unconscious perception and integration of tones. Motives and sources of musical creation and appreciation.
	Metapsychological	– 'non-temporal' correlates and sources of musical experience, **rasanubhava** (aesthesis).

The reversed arrows indicate that **nada anubhava** is a unitary experience and one may start anywhere in this "continuous spectrum".

In the alpha-phoneme theory of **Sankaran** we find an approach that is multiordinal. For limitations of thought processes show, that unless we dispense with thought,

* The attempts of Wheately **Carrington** and others to construct a radically empirical philosophy – 'neutral monism' – to explain the relation of 'mind' and 'matter' cannot be said to be satisfactory in the fulness of experience. So also 'construction of two-dimensional time' by **Broad** & **Price** are ultimately inadequate. The only solution seems **to** be to leave time to itself and recognize the non-temporal wedge, as in the alpha phoneme theory of C. R. **Sankaran**. "The ultimate self, **Dunne** taught, is not in the 'Linear Time' of physics and its Immortality is not to be sought in as an endless prolongation of the time of our emperically describable consciousness". C. **T. K. Chari**, **J. W.** Dunne & the Misconception about Relativistic Time, **The Pilosoph. Quart**. XXIII No. 1, 4. Also, W. CARRINGTON **Matter, Mind and Meaning.** (Methuen).

we shall not be able to 'know' the nature of thought or any such temporal phenomena. Whichever way we proceed, on one end matter and on the other end thought, we find that which is **sui generis** and pervades and sustains all phenomena, but which yet eludes phenomenological definition. It is the admission of this non-temporal wedge that would gives us a fuller understanding.

The alpha-phoneme theory stands on this intuitive truth.

In its comprehensive aspect the theory comprises all 'phenomena" wherever time is involved. So that, though in the beginning the theory was thought of in the context of speech elements like consonants and vowels, it is now extended to any vibratory phenomenon; in the present context it throws light, therefore, on musical problems, more particularly on the mode of vibration of tambura string.

In general terms the theory may be briefly described thus[4] :—

1. Whenever there is a transition from one tone to another, both "must be capable of precise positive definition in mathematical terms."

2. The two tones (analogous to consonant and vowel) are **mutually exclusive only arbitrarily.**

3. From the above, it follows that in the tonal transition, there is no way of positively fixing the termination of one tone and the beginning of the next.

4. As the term tone here includes both production and perception of sound, we have to take into consideration the mode of vibration of a body (strings, vocal chords), the neurophysiological receptors, the psychological process of attention, unconsciousness repressions, etc. We have, therefore, to consider a tonal phenomenon not merely as an acoustical (unidimensional) but as an acoustico - articulatory - psychological (multi-dimensional) continuum.

Now such a tonal- transition is not composed of a few 'discrete' instant phenomena but is an integration of an infinite number of phenomena "differing infinitely little from one another; it is a continuous process. ... these infinite number (of phenomena) differing infinitely little from one another are defined as 'elemental profiles' in the present theory."

In this densely ordered set of elemental profiles no degree of precise experimentation can reveal a change point from one tone to another - i. e., there is no means of saying where one tone ends and the other begins. "At this junction-point, a conceptual limit known as the 'Weierstrass-limit is defined, according to the present theory, in the familiar manner of **Dedekind**'s postulate. This 'conceptual limit' is known as the alpha-region. It is a mind construct."

Analogously the electron jump from one orbit into another as well as the transition from consonant to vowel in human speech are worthy of consideration here.

Obviously this alpha-region is neither one tone nor the other, yet is the progenitor of both. Also, it is not of the order of time; it is the alpha-phoneme - a symbol of experience and not **reason**-able.

However, when temporally we assign an instant of change, it is the moment of temporal generation which is again too 'fleeting' for measurement. Since, yet it is in time - is time itself - it is plenomenological and this 'environment' is the alpha-phonoid of the first order - not measurable but yet existant.

We may, however, recognize an alpha-phonoid of a secondary order. This would be any expedient unit of temporal measurement; its magnitude is entirely a matter of convenience, depending upon what we want to quantise and measure.

The role of the alpha-phoneme theory in music is obvious, where we are concerned with tonal progression in time - physical, physiological and psychological times. We may, therefore, define an environment that can comprehend all these levels. It can be a multi--ordinal environment.

In the particular case of tambura it was observed that there is a periodic frequency change (owing to periodic change in length). It was also shown that the tambura-tone comprises a long retinue of harmonics and inharmonics, **impulsive** in character. There seems to be a very close analogy between the voice and the tambura as pointed out **P. C. Ganeshasundaram**[1]. Such transference and variations of energy are eminently amenable to the methodology of the alphaphoneme theory. Towards this end our future work is directed.

Notes and References :

1. For a preliminary study of the physical phenomena in the tambura see **P. C.** GANESHSUNDARAM, A Note on the Electroacoustic Analysis of the Tambura, (to be published).
2. P H. EDWARDS, A Method for the Quantitative Analysis of Musical Tone, **Phys. Rev.,** p. 37 (1911).
3. A. EHRENZWEIG, **Psychoanalysis of Artistic Vision and Hearing** (Routledge Kegan Paul, 1953).
4. C. R. SANKARAN :
The Problem of the Structure of the Vowels and Consonants in Human Speech **BDCRI**, 9.184 ff (1948).
The Theory of the Alpha-phonoid, **BDCRI** 10.61–4 (1950).
A Contribution to the Study of Speech Stucture, **BDCRI**, 12. nos. 1 & 2 (1952).
Time and Speech-Structure, (with P. C. GANESHSUNDARAM) **op. cit.** Nos. 3&4.
Phonemics of Old Tamil (Decc. Coll. Monogr. No. 7. 1951). In ref. 2 p. 63, ft. nt. 8 he envisages an agreement be-

(continued).

tween speech and music at the hypothetico-deductive level due to the notion of a 'unit-environment'. It is also significant that a close and definable relationship exists between the voice-mechanism (puff theory of SCRIPTURE) and the tambura-string vibration even at the inductive level (vide., ft. nt. 1).

I may suggest here that the unit environment may be 'psychologically' defined at the levels of perception of a physical oscillation as a tone by the mind. For, whatever may be the physical stimulus and the physiological reception, the final integration of these sensations lie with the mind.

Now apart from the problem of 'continuity-discontinuity' of time at the physical and physiological levels (and it is well known that even at the empirical levels both times are not the same) there is the nature of time at 'psychological' levels.

The nature of time at various 'levels' of the mind varies. The unconscious time process is different from the conscious one, as can be seen from the differences in time-experience, say, in dream and waking life. Also, the nature of the collective unconscious – the race memory – and its projections into the individual conscious mind are very significant problems. Similarly, there is a vast difference in the times of the superconscious and conscious processes – as in precognition and cognition. The state of aesthesis (**rasānubhava**) is 'time-less'. Further, even in conscious behaviour attention is not a continuous process.

Again, the switch over from one state to another is a very crucial 'instant' – the change from to sleep to waking, moments of birth and death (non-conscious, conscious transition), **samādhi** to conscious, the 'moment' of aesthesis and etc.

Our understanding of phenomena and non-phenomena must comprise these and more.

In this connection one is reminded of what Prof. REICHENBACH calls the **interphenomena**, namely, 'the unobservable events which happen between the phenomena'. vide., Sir E. WHITTAKER, **From Euclid to Eddington** 84 (Cambridge, 1949).

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Errata

Page	Line	Wrong	Correct
7	Title	Elctro-Acoustic	Electro-Acoustic
12	12	Calculaations	Calculations
13	3, 8th column	as c	as c^2
34	4 right	Sa $= \frac{1}{2}$ Sa	Sa $= \frac{1}{2}$ Sa
	5 right	Sa=2Sa	Sa=2 Sa
	10 left	Sa string Pa string	Sa string Pa string
	11 left	Ma string	Ma string
	13 left	Ma	Ma
	14 left	Sa	Sa
	26 left	Sa string	Sa string
	27 left	Sa $= \frac{1}{2} \times$ Sa	Sa $= \frac{1}{2} \times$ Sa
	33 left	Pa	Pa
	35 left	Pa $= 1\frac{1}{2} \times$ Sa	Pa $= 1\frac{1}{2} \times$ Sa
		Sa	Sa
	36 left	Pa $= 3 \times$ Sa	Pa $= 3 \times$ Sa
	38 left	Sa	Sa
35	20 right	Sa string	Sa string
38	Table	Sa	Sa
		Ma	Ma
		Pa	Pa
39	7	Sa	Sa
		(pa; Ri)	(Pa; Ri)
	9	Ma	Ma
	10	Pa	Pa
		(Ri; dha)	(Ri; Dha)

Page	Line	Wrong	Correct
40	27 right	P^W	P^N
42	5 right	tone	tones
	21 left	explorig	exploring
44	33 right	Sa	Ṣa
45	8 right	⌢⌣	cps
48	1 left	internaal	internal

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